

On the Computation of Topological Indices for Molecular Structures of Subdivided Aztec Diamonds

Muhammad Haroon Aftab^{1,*}, Muhammad Rafaqat¹ and Muhammad Hussain²

¹Department of Mathematics and Statistics, The University of Lahore, Lahore, Pakistan.

Email: haroon.aftab@math.uol.edu.pk, muhammad.rafaqat@math.uol.edu.pk,

²Department of Mathematics, COMSATS University Islamabad, Lahore campus, Pakistan

Email: mhmaths@gmail.com

*Corresponding Author: [e-mail: haroon.aftab@math.uol.edu.pk]

Abstract

In this paper, we will calculate and find new closed formulas using the topological indices such as general Randić index $R_\lambda(\psi)$ for $\lambda = \{1, \frac{1}{2}, -1, -\frac{1}{2}\}$, generalized form of harmonic index, ordinary generalized geometric-arithmetic (OGA) index, sum connectivity index, general sum connectivity index, first & second Gourava indices, first & second hyper-Gourava indices, ABC index, SK index, SK₁ index, SK₂ index, first generalized version of Zagreb index and forgotten index for subdivided aztec diamonds. The discussion on above-mentioned networks will provide us essential outcomes.

Keywords: Vertex degree, Edges, Topological indices, Aztec diamonds.

2020 Mathematics Subject Classification: : 05C09, 05C92

1. Introduction

The branch of mathematics which is related to the implementation of graph theory and chemistry combined is known as "Chemical Graph Theory". It is an essential tool for the study of molecular structure & molecular modeling. It deals with the molecular structure mathematically. Graph theory helps us to recognize the physical properties of chemical structures. In the graph theory, structure of a molecular object is constructed mathematically and labeled on a graph. In this study, atoms are used as vertices and bonds (among atoms) are used as edges. Graph theory offers its essential role in the field of theoretical chemical sciences. Topological index is defined as a numeric parameter of the graph and explains the topology of that graph. We shift the chemical information into the numeric value known as topological indices. Topological indices have substantial parts in QSPR and QSAR study. Above-mentioned topological indices are utilized to confer the bio and physical activities of chemical networks.

D. Vukicević and B. Furtulá presented first (GA) index [1] in 2009 & computed by:

$$GA(\psi) = \sum_{pq \in E(\psi)} \frac{2\sqrt{d_p d_q}}{d_p + d_q}$$

In 2011, an ordinary geometric-arithmetic index of ψ was introduced [2] and computed by for all real k :

$$OGA_k(\psi) = \sum_{pq \in E(\psi)} \left[\frac{\sqrt{4d_p d_q}}{d_p + d_q} \right]^k$$

Kulli introduced the first Gourava index, second Gourava index, first hyper-Gourava index & second hyper-Gourava index [3, 4] in 2017 and calculated by:

$$GO_1(\psi) = \sum_{st \in E(\psi)} [(d_s + d_t) + (d_s d_t)] \&$$

$$GO_2(\psi) = \sum_{st \in E(\psi)} [(d_s + d_t) (d_s d_t)]$$

$$HGO_1(\psi) = \sum_{st \in E(\psi)} [(d_s + d_t) + (d_s d_t)]^2 \&$$

$$HGO_2(\psi) = \sum_{st \in E(\psi)} [(d_s + d_t) (d_s d_t)]^2$$

Randić index [5] was purposed by Milan *Randić* in 1975. It is computed by:

$$R_{\frac{1}{2}}(\psi) = \sum_{pq \in E(\psi)} \frac{1}{\sqrt{(d_p \cdot d_q)}}$$

whereas d_p & d_q are the degrees of the nodes.

With the passage of some time, *Bollöbaś* & *Erdös* presented the general version of *Randić* index called general *Randić* index for λ , where $\lambda \in R$ [6] and computed as:

$$R_\lambda(\psi) = \sum_{pq \in E(\psi)} (d_p \cdot d_q)^\lambda, \text{ for } \lambda = \{-1, 1, -\frac{1}{2}, \frac{1}{2}\}$$

L. Zhong presented the harmonic index [7] in 2012 and calculated by:

$$HI(\psi) = \sum_{pq \in E(\psi)} \frac{2}{d_p + d_q}$$

In 2015, L. Yan presented the generalized form of the harmonic index [8] and evaluated by:

$$H_k I(\psi) = \sum_{pq \in E(\psi)} \left[\frac{2}{d_p + d_q} \right]^k$$

Ernestö Estrada et al. [9] presented a new descriptor, called ABC index in 2008 and evaluated by:

$$ABC(\psi) = \sum_{pq \in E(\psi)} \sqrt{\frac{d_p + d_q - 2}{d_p d_q}}$$

For a graph ψ , the SK, SK_1 , SK_2 [10], the indices can be computed as:

$$SK(\psi) = \sum_{pq \in E(\psi)} \frac{d_p + d_q}{2},$$

$$SK_1(\psi) = \sum_{pq \in E(\psi)} \frac{d_p d_q}{2} \&$$

$$SK_2(\psi) = \sum_{pq \in E(\psi)} \left[\frac{d_p + d_q}{2} \right]^2$$

Supposing d_p & d_q are the degrees of nodes p & q in ψ , respectively.

B. Lučić presented the sum connectivity index G [11] in 2009 and evaluated by:

$$G_{-\frac{1}{2}}(\psi) = \sum_{pq \in E(\psi)} [d_p + d_q]^{-\frac{1}{2}}$$

The generalized version of sum connectivity index was presented by Trinajstić and Zhou in 2010 [12, 13].

$$G_k(\psi) = \sum_{pq \in E(\psi)} [d_p + d_q]^k$$

Li & Zheng purposed the general version of first Zagreb index in 2005 [14] and calculated as:

$$^kM_1(\psi) = \sum_{pq \in E(\psi)} [d_p^{k-1} + d_q^{k-1}], \quad k \in \mathbb{R}, \quad k \neq 0 \& \quad k \neq 1$$

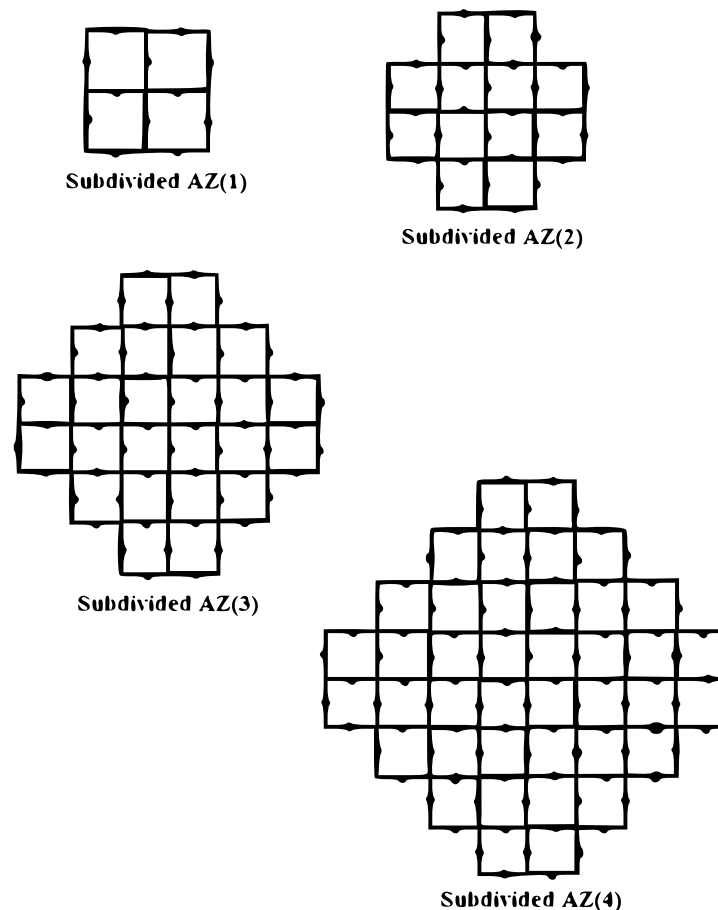
Furtulá & Gutman invented a new index called F-index [15] in 2015 and formulated by:

$$F(\psi) = \sum_{pq \in E(\psi)} [d_p^2 + d_q^2]$$

2. Results for Degree based Topological indices for Subdivided Aztec diamonds

An n -dimensional Aztec diamonds comprises of all squares lattices centered at (x, y) satisfying the condition $|x - \frac{1}{2}| + |y - \frac{1}{2}| \leq n$, these squares are unit squares, and the region obtained in this way is the union of all unit squares. Furthermore, we introduce a new vertex of degree 2 at each of the edges of the unit square to obtain a subdivided Aztec diamonds. In this part, we will calculate the degree-based topological indices that are mentioned above to find some closed formulas for subdivided aztec diamonds network. By supposing that ψ is the graph of the subdivided aztec diamonds network SAZ_n , we have the order of SAZ_n , $|V(\psi)| = 6n^2 + 14n + 1$ and the size of SAZ_n , $|E(\psi)| = 8n^2 + 16n$. There are three different kinds of edges $(2, 2)$, $(2, 3)$, & $(2, 4)$.

Figure 2.1 shows the various dimensions of the subdivided Aztec diamonds. We define here two necessary objects called nodes and edges for the graph shown in figure 2.1 and symbolized by ψ . Topological indices which are purely degree based are computed for the subdivided Aztec diamonds networks. In the following theorems, various formulas of the above-mentioned indices for subdivided aztec diamonds SAZ_n are computed. Let d_p be the degree of vertex & belongs to $V(\psi)$ and d_q be the degree of edge & belongs to $E(\psi)$ and $e = |E(\psi)|$ is the edge cardinality of ψ . The SAZ_n is based on the end nodes of each edge.

Figure 2.1: SAZ_4 **Table-1:** Edges-separation of a graph ψ created from the degree of end vertices for all edges.

(d_p, d_q) for $pq \in E(\psi)$	Number of $E(\psi)$
$(2, 2)$	$8n$
$(2, 3)$	12
$(2, 4)$	$8n^2 + 8n - 12$

Theorem 2.1 Consider SAZ_n as a subdivided Aztec diamonds network, then its OGA index can be computed as:

$$OGA_k(\psi) = 8n^2 \left[\frac{\sqrt{32}}{6} \right]^k + 8n \left\{ \left[\frac{\sqrt{32}}{6} \right]^k + 1 \right\} + 12 \left\{ \left[\frac{\sqrt{24}}{5} \right]^k - \left[\frac{\sqrt{32}}{6} \right]^k \right\}.$$

Proof: By supposing ψ is the subdivided aztec diamonds network SAZ_n , from table-1, getting

$$OGA_k(\psi) = \sum_{pq \in E(\psi)} \left[\frac{\sqrt{4d_p d_q}}{d_p + d_q} \right]^k$$

$$OGA_k(\psi) = (8n) \left[\frac{\sqrt{16}}{2+2} \right]^k + (12) \left[\frac{\sqrt{24}}{5} \right]^k + (8n^2 + 8n - 12) \left[\frac{\sqrt{32}}{6} \right]^k$$

By making some computations, we get

$$OGA_k(\psi) = 8n^2 \left[\frac{\sqrt{32}}{6} \right]^k + 8n \left\{ \left[\frac{\sqrt{32}}{6} \right]^k + 1 \right\} + 12 \left\{ \left[\frac{\sqrt{24}}{5} \right]^k - \left[\frac{\sqrt{32}}{6} \right]^k \right\}.$$

Theorem 2.2 Consider SAZ_n as a subdivided Aztec diamonds network, then its first Gourava index and second Gourava index are computed as:

$$GO_1(\psi) = 112n^2 + 176n - 36 \quad \text{and} \quad GO_2(\psi) = 384n^2 + 512n - 216.$$

Proof:- By supposing ψ is the subdivided aztec diamonds network SAZ_n . From table-1, getting

$$GO_1(\psi) = \sum_{st \in E(\xi)} [(d_s + d_t) + (d_s d_t)] \quad \&$$

$$GO_2(\psi) = \sum_{st \in E(\psi)} [(d_s + d_t)(d_s d_t)]$$

After some calculations, we have

$$GO_1(\psi) = (8n)[(8)] + (12)[11] + (8n^2 + 8n - 12)[14] = 112n^2 + 176n - 36 \quad \text{and} \quad GO_2(\psi) = (8n)[16] + (12)[30] + (8n^2 + 8n - 12)[48] = 384n^2 + 512n - 216.$$

Theorem 2.3 Consider SAZ_n as a subdivided Aztec diamonds network, then its first hyper-Gourava index and second hyper-Gourava index are computed as:

$$HGO_1(\psi) = 1568n^2 + 2080n - 900 \quad \text{and} \quad HGO_2(\psi) = 18432n^2 + 20480n - 16848.$$

Proof:- By supposing ψ is the subdivided aztec diamonds network, SAZ_n . From table-1, getting

$$HGO_1(\psi) = \sum_{st \in E(\psi)} [(d_s + d_t) + (d_s d_t)]^2 \quad \&$$

$$HGO_2(\psi) = \sum_{st \in E(\psi)} [(d_s + d_t)(d_s d_t)]^2$$

By making some computations, we get

$$HGO_1(\psi) = (8n)[64] + (12)[121] + (8n^2 + 8n - 12)[196] = 1568n^2 + 2080n - 900 \quad \text{and}$$

$$HGO_2(\psi) = (8n)[256] + (12)[900] + (8n^2 + 8n - 12)[2304] = 18432n^2 + 20480n - 16848.$$

Theorem 2.4 Consider SAZ_n as a subdivided Aztec diamonds network, then its general Randić index is calculated as:

$$R_\lambda(SAZ_n) = \begin{cases} n^2 + 3n + \frac{1}{2} & \text{for } \lambda = -1 \\ \sqrt{8}n^2 + n(4 + \sqrt{8}) + (2\sqrt{6} - 3\sqrt{2}) & \text{for } \lambda = -\frac{1}{2} \\ 8\sqrt{8}n^2 + 8n(2 + \sqrt{8}) + 12(\sqrt{6} - \sqrt{8}) & \text{for } \lambda = \frac{1}{2} \\ 64n^2 + 96n - 24 & \text{for } \lambda = 1 \end{cases}$$

Proof:- By supposing ψ is the subdivided aztec diamonds network SAZ_n of n dimensions. We have number of nodes & edges in SAZ_n , $|V(SAZ_n)| = 6n^2 + 14n + 1$ and $|E(SAZ_n)| = 8n^2 + 16n$, respectively.

We know that

$$R_\lambda(\psi) = \sum_{pq \in E(\psi)} [d_p \cdot d_q]^\lambda \text{ ----- (1) for } \lambda = \{-1, 1, -\frac{1}{2}, \frac{1}{2}\}$$

Case-1: For $\lambda = -1$, its Randić' index $R_\lambda(\psi)$ can be computed as:

$$R_{-1}(\psi) = \sum_{pq \in E(\psi)} \frac{1}{d_p d_q} \quad \text{Using (1), from table-1, we get}$$

$$R_{-1}(\psi) = (2n) + (2) + (8n^2 + 8n - 12)(8)^{-1}$$

After some computations, we get

$$R_{-1}(\psi) = n^2 + 3n + \frac{1}{2}.$$

Case-2: For $\lambda = -\frac{1}{2}$, its Randić' index $R_\lambda(\psi)$ can be computed as:

$$R_{-\frac{1}{2}}(\psi) = \sum_{pq \in E(\psi)} \frac{1}{\sqrt{(d_p \cdot d_q)}} \quad \text{Using (1), from table-1, we know}$$

$$R_{-\frac{1}{2}}(\psi) = (8n)\frac{1}{\sqrt{4}} + (12)\frac{1}{\sqrt{6}} + (8n^2 + 8n - 12)\frac{1}{\sqrt{8}}$$

After some computations, we get

$$R_{-\frac{1}{2}}(\psi) = \sqrt{8}n^2 + n(4 + \sqrt{8}) + (2\sqrt{6} - 3\sqrt{2}).$$

Case-3: For $\lambda = \frac{1}{2}$, its Randić' index $R_\lambda(\psi)$ can be computed as:

$$R_{\frac{1}{2}}(\psi) = \sum_{pq \in E(\psi)} \sqrt{(d_p \cdot d_q)} \quad \text{Using (1), from table-1, getting}$$

$$R_{\frac{1}{2}}(\psi) = (8n)\sqrt{4} + (12)\sqrt{6} + (8n^2 + 8n - 12)\sqrt{8}$$

By doing some calculations, we get

$$R_{\frac{1}{2}}(\psi) = 8\sqrt{8}n^2 + 8n(2 + \sqrt{8}) + 12(\sqrt{6} - \sqrt{8}).$$

Case-4: For $\lambda = 1$, its Randić' index $R_\lambda(\psi)$ can be computed as:

$R_1(\psi) = \sum_{pq \in E(\psi)} (d_p d_q)^1$ Using (1), from table-1, we know

$$R_1(\psi) = (8n)(4) + (12)(6) + (8n^2 + 8n - 12)(8)$$

By doing some calculations, we get $R_1(\psi) = 64n^2 + 96n - 24$.

Theorem 2.5 Consider SAZ_n as a subdivided Aztec diamonds network, then its harmonic index is computed as:

$$HI(\psi) = \frac{4}{15} \{10n^2 + 25n + 3\}.$$

Proof:- By supposing ψ is the subdivided aztec diamonds network SAZ_n . From table-1, getting

$$HI(\psi) = \sum_{pq \in E(\psi)} \frac{2}{d_p + d_q}$$

After simplifications, we obtain,

$$HI(\psi) = (8n) \left[\frac{1}{2} \right] + (12) \left[\frac{2}{5} \right] + (8n^2 + 8n - 12) \left[\frac{1}{3} \right] = \frac{1}{15} \{40n^2 + 100n + 12\}.$$

Theorem 2.6 Consider SAZ_n as a subdivided Aztec diamonds network, then its generalized form of harmonic index is computed as:

$$H_k I(\psi) = \frac{8n^2 \{10^k\} + 8n \{15^k + 10^k\} + 12 \{12^k - 10^k\}}{30^k}.$$

Proof:- By supposing ψ is the subdivided aztec diamonds network SAZ_n . From table-1, getting

$$H_k I(\psi) = \sum_{pq \in E(\psi)} \left[\frac{2}{d_p + d_q} \right]^k$$

By making some computations, we get

$$\begin{aligned} H_k I(\psi) &= (8n) \left[\frac{1}{2} \right]^k + (12) \left[\frac{2}{5} \right]^k + (8n^2 + 8n - 12) \left[\frac{2}{6} \right]^k \\ &= \frac{8n^2 \{10^k\} + 8n \{15^k + 10^k\} + 12 \{12^k - 10^k\}}{30^k}. \end{aligned}$$

Theorem 2.7 Consider SAZ_n as a subdivided Aztec diamonds network, then its ABC index is computed as:

$$ABC(\psi) = \sqrt{2}n[4n + 8].$$

Proof:- By supposing ψ is the subdivided aztec diamonds network SAZ_n . From table-1, getting

$$ABC(\psi) = \sum_{pq \in E(\psi)} \sqrt{\frac{d_p + d_q - 2}{d_p d_q}}$$

By making some computations, we get

$$\begin{aligned} ABC(\psi) &= (8n) \sqrt{\frac{2+2-2}{4}} + (12) \sqrt{\frac{2+3-2}{6}} + (8n^2 + 8n - 12) \sqrt{\frac{2+4-2}{8}} \\ &= 4\sqrt{2}n[n + 2]. \end{aligned}$$

Theorem 2.8 Consider SAZ_n as a subdivided Aztec diamonds network, then its SK index, SK_1 index, and SK_2 index are calculated as:

$$SK(\psi) = 24n^2 + 40n - 6, \quad SK_1(\psi) = 32n^2 + 48n - 12 \text{ \&}$$

$$SK_2(\psi) = 72n^2 + 104n - 33, \text{ respectively.}$$

Proof:- By supposing ψ is the subdivided aztec diamonds network SAZ_n . From table-1, getting

$$SK(\psi) = \sum_{pq \in E(\psi)} \frac{d_p + d_q}{2}, \quad SK_1(\xi) = \sum_{pq \in E(\xi)} \frac{d_p d_q}{2} \text{ \&}$$

$$SK_2(\psi) = \sum_{pq \in E(\psi)} \left[\frac{d_p + d_q}{2} \right]^2$$

By making some calculations, we get

$$SK(\psi) = (8n)(2) + (12) \left(\frac{5}{2} \right) + (8n^2 + 8n - 12)(3) = 24n^2 + 40n - 6.$$

$$SK_1(\psi) = (8n)(2) + (12)(3) + (8n^2 + 8n - 12)(4) = 32n^2 + 48n - 12.$$

$$SK_2(\psi) = (8n)(4) + (12) \left(\frac{25}{4} \right) + (8n^2 + 8n - 12)(9) = 72n^2 + 104n - 33.$$

Theorem 2.9 Consider SAZ_n as a subdivided Aztec diamonds network, then its sum connectivity index is calculated as:

$$G_{-\frac{1}{2}}(\psi) = n^2 \left[\frac{8}{\sqrt{6}} \right] + 4n \left[1 + \frac{2}{\sqrt{6}} \right] + 12 \left[\frac{1}{\sqrt{5}} - 1 \right].$$

Proof:- By supposing G is the subdivided aztec diamonds network SAZ_n . From table-1, getting

$$G_{-\frac{1}{2}}(\psi) = \sum_{pq \in E(\psi)} [d_p + d_q]^{-\frac{1}{2}}$$

$$G_{-\frac{1}{2}}(\psi) = (8n) \left(\frac{1}{2} \right) + (12) \left(\frac{1}{\sqrt{5}} \right) + (8n^2 + 8n - 12) \left(\frac{1}{\sqrt{6}} \right)$$

By making some computations, we get

$$G_{-\frac{1}{2}}(\psi) = n^2 \left[\frac{8}{\sqrt{6}} \right] + 4n \left[1 + \frac{2}{\sqrt{6}} \right] + 12 \left[\frac{1}{\sqrt{5}} - 1 \right].$$

Theorem 2.10 Consider SAZ_n as a subdivided Aztec diamonds network, then its general sum connectivity index is calculated as:

$$G_k(\psi) = 8n^2 [6^k] + 8n [6^k + 4^k] + 12 [5^k - 6^k].$$

Proof:- By supposing ψ is the subdivided aztec diamonds network SAZ_n . From table-1, getting

$$G_k(\psi) = \sum_{pq \in E(\psi)} [d_p + d_q]^k$$

$$G_k(\psi) = (8n) [4^k] + (12) [5^k] + (8n^2 + 8n - 12) [6^k]$$

By making some computations, we get

$$G_k(\psi) = 8n^2 [6^k] + 8n [6^k + 4^k] + 12 [5^k - 6^k].$$

Theorem 2.11 Consider SAZ_n as a subdivided Aztec diamonds network, then its first general Zagreb index is computed as:

$${}^kM_1(\psi) = 2n^2 [2^{k+1} + 4^k] + 2n [4^k + 6(2^k)] + [4(3^k) - 3(4^k)].$$

Proof:- By supposing ψ is the subdivided aztec diamonds network SAZ_n . From table-1, getting

$${}^kM_1(\psi) = \sum_{pq \in E(\psi)} [d_p^{k-1} + d_q^{k-1}], \quad k > 1$$

$${}^kM_1(\psi) = (8n) [2^k] + (12) [2^{k-1} + 3^{k-1}] + (8n^2 + 8n - 12) [2^{k-1} + 4^{k-1}]$$

By making some computations, we get

$${}^kM_1(\psi) = 2n^2 [2^{k+1} + 4^k] + 2n [4^k + 6(2^k)] + [4(3^k) - 3(4^k)].$$

Theorem 2.12 Consider SAZ_n as a subdivided Aztec diamonds network, then its F-index is computed as:

$$F(\psi) = 160n^2 + 224n - 84.$$

Proof:- By supposing ψ is the subdivided aztec diamonds network SAZ_n . From table-1, getting

$$F(\psi) = \sum_{pq \in E(\psi)} [d_p^2 + d_q^2]$$

$$F(\psi) = (8n) (8) + (12) (13) + (8n^2 + 8n - 12) (20)$$

By making some computations, we get

$$F(\psi) = 160n^2 + 224n - 84.$$

3. Conclusions

In this article, many topological invariants which are purely degree-based like general *Randić* index $R_\lambda(\psi)$ for $\lambda = \{1, \frac{1}{2}, -1, -\frac{1}{2}\}$, general version of harmonic index, ordinary generalized geometric-arithmetic(OGA) index, sum connectivity index, general sum connectivity index, first Gourava index and second Gourava index, first hyper-Gourava index and second hyper-Gourava index, ABC index, different SK's indices, F-index and others indices for subdivided aztec diamonds are discussed. The above computed indices can be interlinked with the chemical networks to grasp their bio-physical features.

Acknowledgement

All authors are highly grateful to the precious remarks and critiques of the referees that helped this manuscript in its nurturing.

References

- [1] D. Vukičević and B. Furtula. Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges. *Journal of Mathematical Chemistry*, 46(4): 1369-1376.
- [2] Mehdi Eliasi, Ali Iranmanesh, On ordinary generalized geometric–arithmetic index, *Applied Mathematics Letters*, Volume 24, Issue 4, 2011, Pages 582-587, ISSN 0893-9659.
- [3] V. R. Kulli, The Gourava indices and coindices of Graphs. *Annals of Pure and Applied Mathematics*. 14(1), 2017, 33-38.
- [4] V. R. Kulli, On hyper-Gourava indices and coindices. *International Journal of Mathematical Archive*. 8(12), 2017, 116-120.
- [5] M. Randić, On characterization of molecular branching, *J. Amer. Chem. Soc.* 97 (1975), 6609–6615.
- [6] B. Bollobás, P. Erdős: Graphs of extremal weights. *Ars Comb.* 50, 225-233 (1998).
- [7] S. Fajtlowicz, On conjectures of Graffiti-II, *Congressus Numerantium*.
- [8] L. Yan, W. Gao, J. Li, General Harmonic Index and General Sum Connectivity Index of Polyomino Chains and Nanotubes. *Journal of Computational and Theoretical Nanoscience*. 12(10), 2015, 3940-3944.
- [9] E. Estrada, L. Torres, L. Rodríguez, I. Gutman, An atom-bond connectivity index: Modelling the enthalpy of formation of alkanes. 1998, *Indian J. Chem.* 37A, 849-855.
- [10] V. S. Shegehalli and R. Kanabur, Computation of New Degree-Based Topological Indices of Graphene, *Journal of Mathematics*, vol. 5, 2016, Article Id 4341919, 6 pages.
- [11] B. Lučić, N. Trinajstić, and B. Zhou, Comparison between the sum-connectivity index and product-connectivity index for benzenoid hydrocarbons. *Chemical Physics Letters*, 475(1-3): 146-148.
- [12] B. Zhou, N. Trinajstić, On General Sum Connectivity Index. *Journal of Mathematical Chemistry*. 47, 2010, 210-218.
- [13] L. Yan, W. Gao, and J. Li. General harmonic index and general sum connectivity index of polyomino chains and nanotubes. *Journal of Computational and Theoretical Nanoscience*, 12(10): 3940-3944.
- [14] X. Li and J. Zhend, A unified approach to the extremal trees for different indices, *MATCH Commun. Math. Comput. Chem.*
- [15] B. Furtula and I. Gutman. A forgotten topological index. *Journal of Mathematical Chemistry*, 53(4): 1184-1190.