



Multi-Criteria Group Decision Making Method TOPSIS for Interval-Valued 2-Tuple Linguistic Information

Tabasam Rashid*

¹University of Management and Technology, Lahore-54770, Pakistan. tabasam.rashid@gmail.com

*Corresponding Author: Tabasam Rashid

Abstract

Technique for order preference by similarity to ideal solution is proposed for multi-criteria group decision making problems based on interval-valued 2-tuple linguistic information. Interaction weights amongst decision makers are evaluated by Choquet integral and interval-valued 2-tuple weighted averaging operator is used to aggregate the opinion of decision makers. Metric is defined for interval-valued 2-tuple linguistic information. For the complete elaboration of the proposed method an interesting example is given the selection of suitable alternative.

Keywords: Multi attribute group decision making; 2-tuple model; aggregation operator; TOPSIS.

Mathematics Subject Classification: 2010 : 62C86; 90B50; 91B06; 03B52; 62A86.

1. Introduction

Presentation of information and to take serious decisions on the basis of these information is always difficult. Hwang and Yoon played a great role to develop a method to solve multi-criteria decision making problems by technique for order preference by similarity to ideal solution (TOPSIS) in [14]. TOPSIS is a useful and practical technique for the selection and ranking of alternatives on the basis of several criteria. Final selected alternative using TOPSIS have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. In this technique, ordinary/crisp values were used for weights and performance ratings of the involved criteria in the decision problem. Ambiguity in information is almost always exist. Judgment and preference given by human beings are often ambiguous and cannot be estimated with exact numeric values, so to model the real world decision making problems crisp values are not suitable. To handle uncertainty, ordinary set theory extended to fuzzy set theory by Zadeh in 1965 [31]. This theory has been successfully used to handle imprecision in decision making problems. TOPSIS is modified for several extensions of fuzzy set and used in real life applications [8, 19, 21, 22].

2-tuple fuzzy linguistic representation model was developed in [11]. This model based on linguistic term and numeric value. This information model has been successfully applied in several decision making problems [3, 6, 7, 18, 20, 24, 26] and some popular aggregation operators have been developed to use in decision problems. Herrera and Martinez proposed the arithmetic averaging operator, the

arithmetic weighted averaging operator, the ordered weighted averaging operator and the extended weighted averaging operator in [12] for the case of 2-tuple linguistic model. Jiang and Fan studied the 2-tuple ordered weighted averaging operator and the 2-tuple ordered weighted geometric operator in [15]. Zhang and Fan extended the 2-tuple ordered weighted averaging operator with several characteristics of this operator [32]. In all these mentioned aggregation operators, the attributes are used to be independent of each other and independent axiom is used to characterize them [23]. But most of the time in the real/practical life decision making problems, the attributes involved in the decision problems are often correlated or inter-dependent. This correlation or inter-dependence amongst the elements carefully managed by Choquet integral [4]. This integral successfully studied and applied in several decision making methods to handle the correlation amongst the alternatives and attributes [2, 5, 9, 13, 27]. Yager used the Choquet integral to order the collective real arguments by defining the induced Choquet ordered averaging aggregation operator in [28]. Choquet integral has been used extensively in the field of aggregation operators. Meyer [17] used this integral to aggregate the fuzzy numbers and he also proposed the fuzzy version of this popular integral. It was important to aggregated the 2-tuple linguistic information model by taking care of the inter-dependence of alternatives and attributes in the aggregation. Yang and Chen defined some aggregation operators based on Choquet integral for the case of 2-tuple model to handle the dependence among the alternatives [29]. We refer to the book of Martínez et. al [16] for the basics on 2-tuple linguistic model. The extension or generalization of 2-tuple model to interval-valued 2-tuple (IV2T) fuzzy linguistic information was given by Beg and Rashid in [1]. They also developed IV2T aggregation operators based on Choquet integral. Furthermore, they proposed a multiple attribute decision making method by using these operators. An extended VIKOR method developed for group multi-criteria supplier selection with interval 2-tuple linguistic information [30]. Still, we do not have TOPSIS for IV2T linguistic information. In this research, this technique is modified for the IV2T linguistic information.

This paper is structured as follows: some basic notions and preliminary concepts are discussed in Section 2. In Section 3, some new operations between IV2T linguistic arguments are defined and distance measure between any two IV2T linguistic arguments is proposed. In Section 4, TOPSIS is proposed to solve a group decision making problem for IV2T linguistic information. In Section 5, an illustrative example is discussed in detail to illustrate the proposed method. In Section 6, some conclusions are given.

2. Basic Concepts

Suppose that $S = \{s_i | i = 1, \dots, t\}$ is a totally ordered, finite and discrete term set, where s_i is a possible linguistic term for a linguistic variable.

For example; $S = \{s_1=\text{extremely poor (EP)}, s_2=\text{very poor (VP)}, s_3=\text{poor (P)}, s_4=\text{medium (M)}, s_5=\text{good (G)}, s_6=\text{very good (VG)}, s_7=\text{extremely good (EG)}\}$.

The above set satisfies the following properties:

- (1) The set is ordered: $s_i \geq s_j$, if $i \geq j$;
- (2) The max operator: $\max(s_i, s_j) = s_i$, if $i \geq j$;
- (3) The min operator: $\min(s_i, s_j) = s_i$, if $i \leq j$.

The IV2T $(s_i, [a_i, b_i])$ is used to represent the linguistic information, where s_i is a linguistic label from a predefined linguistic term set S and $[a_i, b_i]$ is a numerical interval-valued representing the value of the possible symbolic translation and $[a_i, b_i] \in D[i - 0.5, i + 0.5)$, where $D[i - 0.5, i + 0.5)$ is a collection of all closed subintervals of $[i - 0.5, i + 0.5)$.

Definition 2.1. [25] Fuzzy measure μ on the set X is defined as $\mu : P(X) \rightarrow [0, 1]$ satisfying the following axioms:

1. $\mu(\emptyset) = 0, \mu(X) = 1$;
2. $B \subseteq C$ implies $\mu(B) \leq \mu(C)$, for all $B, C \subseteq X$;
3. $\mu(B \cup C) = \mu(B) + \mu(C) + \rho \mu(B)\mu(C)$ for all $B, C \subseteq X$ and $B \cap C = \emptyset$, where $\rho \in (-1, +\infty)$.

Here, the value of parameter ρ is used to handle the interaction between criteria.

If X is a finite set, then $\bigcup_{i=1}^n x_i = X$. The ρ -fuzzy measure μ satisfies following Eq. (2.1)

$$\mu(X) = \mu\left(\bigcup_{i=1}^n x_i\right) = \begin{cases} \frac{1}{\rho} \left\{ \prod_{i=1}^n [1 + \rho \mu(x_i)] - 1 \right\} & \text{if } \rho \neq 0, \\ \sum_{i=1}^n \mu(x_i) & \text{if } \rho = 0, \end{cases} \quad (2.1)$$

where $x_i \cap x_j = \emptyset$ for all $i, j = 1, 2, \dots, n$ and $i \neq j$.

For any subset A of X , we have

$$\mu(A) = \begin{cases} \frac{1}{\rho} \left\{ \prod_{x_i \in A} [1 + \rho \mu(x_i)] - 1 \right\} & \text{if } \rho \neq 0, \\ \sum_{x_i \in A} \mu(x_i) & \text{if } \rho = 0. \end{cases} \quad (2.2)$$

Based on Eq. (2.1), the value of ρ can be uniquely determined from $\mu(X) = 1$, which is equivalent to solving

$$1 = \frac{1}{\rho} \left\{ \prod_{i=1}^n [1 + \rho \mu(x_i)] - 1 \right\} \quad (2.3)$$

If the elements of B in X are independent, we have

$$\mu(B) = \sum_{x_i \in B} \mu(x_i), \text{ for any } B \subseteq X. \quad (2.4)$$

Definition 2.2. [1] Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S , i.e., the result of a symbolic aggregation operation, $\left[\frac{\beta \in}{1, t}\right]$ with t being the cardinality of S . Let $i = \lim \text{funcround}(\beta)$, here $\lim \text{funcround}(\cdot)$ is the usual round operation, and

$$\alpha = \begin{cases} [i, \beta], & \text{if } i \leq \beta, \\ [\beta, i], & \text{if } i \geq \beta, \end{cases}$$

such that $i \in \{1, 2, \dots, t\}$ and $\alpha \in D[i - 0.5, i + 0.5)$, then α is called a possible symbolic translation.

Definition 2.3. [1] Let S be a linguistic term set and β be a number representing the aggregation result of linguistic symbolic. The function $\Delta : [1, t + 1) \rightarrow S \times D[i - 0.5, i + 0.5)$, used to obtain the IV2T linguistic information equivalent to β is defined as $\Delta(\beta) = (s_i, \alpha)$ where

$$i = \begin{cases} \lim \text{funcround}(\beta) & \text{if } \lim \text{funcround}(\beta) \leq t \\ t & \text{if } \lim \text{funcround}(\beta) > t \end{cases}$$

and

$$\alpha = \begin{cases} [i, \beta], & \text{if } i \leq \beta \text{ and } \lim funcround(\beta) \leq t, \\ [i, t + 0.49], & \text{if } i \leq \beta \text{ and } \lim funcround(\beta) > t, \\ [\beta, i], & \text{if } i \geq \beta, \end{cases}$$

where s_i has the closest index label to β and α is the interval of the possible symbolic translation.

Definition 2.4. [1] Let S be a linguistic term set, $(s_i, [a, b])$ be a IV2T. The function ∇ from a IV2T $(s_i, [a, b])$ to its numerical value $\beta \in [-0.5, t + 0.5] \subset \mathbb{R}$ can be defined as follows:

$$\begin{aligned} \nabla : S \times D[i - 0.5, i + 0.5] &\rightarrow [1, t + 1), \\ \nabla(s_i, [a, b]) &= i + (b - a) = \beta. \end{aligned}$$

Let $X = \{x_1, x_2, \dots, x_n\}$ be the set of the attributes, $P(X)$ be the power set of X and $\mu(x_i)$ ($i = 1, 2, \dots, n$) be the weight of the elements $x_i \in X$, where μ is a fuzzy measure.

Definition 2.5. [1] Let $(r_1, [a_1, b_1]), (r_2, [a_2, b_2]), \dots, (r_n, [a_n, b_n])$ be IV2T linguistic arguments, X be the set of attributes and μ be a fuzzy measure on X , then the IV2T correlated averaging (ITCA) operator is defined as follows:

$$\begin{aligned} &ITCA_{\mu}((r_1, [a_1, b_1]), (r_2, [a_2, b_2]), \dots, (r_n, [a_n, b_n])) \\ &= \Delta \left(\sum_{i=1}^n (\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})) \nabla(r_{\sigma(i)}, [a_{\sigma(i)}, b_{\sigma(i)}]) \right), \end{aligned}$$

here $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(1, 2, \dots, n)$ such that $(r_{\sigma(1)}, [a_{\sigma(1)}, b_{\sigma(1)}]) \geq (r_{\sigma(2)}, [a_{\sigma(2)}, b_{\sigma(2)}]) \geq \dots \geq (r_{\sigma(n)}, [a_{\sigma(n)}, b_{\sigma(n)}])$, where $x_{\sigma(i)}$ is the attribute corresponding to $(r_{\sigma(i)}, [a_{\sigma(i)}, b_{\sigma(i)}])$, $H_{\sigma(i)} = \{x_{\sigma(k)} | k \leq i\}$ for $i \geq 1$, $H_{\sigma(0)} = \emptyset$.

For a special case of the ITCA operator, let $(r_{\sigma(i)}, [a_{\sigma(i)}, b_{\sigma(i)}])$ ($i = 1, 2, \dots, n$) be IV2T linguistic arguments.

If the dependent condition (2.4) holds, then

$$\mu(x_{\sigma(i)}) = \mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)}), i = 1, 2, \dots, n. \quad (2.5)$$

In this situation, the ITCA operator reduces to the following IV2T weighted averaging (ITWA) operator:

$$ITWA_{\mu}((r_1, [a_1, b_1]), (r_2, [a_2, b_2]), \dots, (r_n, [a_n, b_n])) = \Delta \left(\sum_{i=1}^n \mu(x_i) \nabla(r_{\sigma(i)}, [a_{\sigma(i)}, b_{\sigma(i)}]) \right).$$

3. New operations between IV2T linguistic arguments

Some new operations between IV2T linguistic arguments are defined to develop TOPSIS for multi-criteria decision making problems. Since distance plays a major role in TOPSIS, so the distance measure between any two IV2T linguistic arguments is given in this section.

Definition 3.1. Let $(s_i, [a_i, b_i])$ and $(s_j, [a_j, b_j])$ be two IV2Ts, they have the following properties:

1. If $s_i < s_j$ then we say $(s_i, [a_i, b_i])$ is smaller than $(s_j, [a_j, b_j])$,
2. If $s_i = s_j$, and;
 - i. $b_i < b_j$, then we say $(s_i, [a_i, b_i])$ is smaller than $(s_j, [a_j, b_j])$;

- ii. $b_i = b_j$ and $a_i < a_j$, then we say $(s_i, [a_i, b_i])$ is smaller than $(s_j, [a_j, b_j])$.
- iii. $b_i = b_j$ and $a_i = a_j$, then we say $(s_i, [a_i, b_i]) = (s_j, [a_j, b_j])$.

Definition 3.2. max and min operators for IV2T linguistic arguments are defined as;

$$\begin{aligned} & \max((r_1, [a_1, b_1]), (r_2, [a_2, b_2]), \dots, (r_n, [a_n, b_n])) \\ &= (\max(r_1, r_2, \dots, r_n), [\max(a_1, a_2, \dots, a_n), \max(b_1, b_2, \dots, b_n)]) \\ & \min((r_1, [a_1, b_1]), (r_2, [a_2, b_2]), \dots, (r_n, [a_n, b_n])) \\ &= (\min(r_1, r_2, \dots, r_n), [\min(a_1, a_2, \dots, a_n), \min(b_1, b_2, \dots, b_n)]) \end{aligned}$$

Definition 3.3. Distance between any two IV2T linguistic arguments is defined as follows:

$$D((r_i, [a, b]), (r_j, [c, d])) = |i - j| + |a - c| + |b - d|$$

Proposition 3.4. Distance defined in definition 3.3 satisfies the following four conditions.

1. $D((r_i, [a, b]), (r_j, [c, d])) = 0$ iff $(r_i, [a, b]) = (r_j, [c, d])$
2. $D((r_i, [a, b]), (r_j, [c, d])) \geq 0$.
3. $D((r_i, [a, b]), (r_j, [c, d])) = D((r_j, [c, d]), (r_i, [a, b]))$
4. $D((r_i, [a, b]), (r_j, [c, d])) + D((r_j, [c, d]), (r_k, [e, f])) \geq D((r_i, [a, b]), (r_k, [e, f]))$

Proof. 1. $D((r_i, [a, b]), (r_j, [c, d])) = 0$

$$\iff |i - j| + |a - c| + |b - d| = 0$$

$$\iff i = j, a = c \text{ and } b = d$$

$$\iff s_i = s_j, a = c \text{ and } b = d$$

$$\iff (r_i, [a, b]) = (r_j, [c, d]).$$

2. Since $i, j \in \mathbb{N}$ and a, b, c, d are positive real numbers, then $|i - j| \geq 0$, $|a - c| \geq 0$ and $|b - d| \geq 0$. As we know that the sum of non-negative numbers is also non-negative, so $|i - j| + |a - c| + |b - d| \geq 0$.

Thus, $D((r_i, [a, b]), (r_j, [c, d])) \geq 0$.

3. Since $D((r_i, [a, b]), (r_j, [c, d])) = |i - j| + |a - c| + |b - d|$ and $|x - y| = |y - x|$ for any $x, y \in \mathfrak{R}$. So, $D((r_i, [a, b]), (r_j, [c, d])) = |j - i| + |c - a| + |d - b|$
Hence $D((r_i, [a, b]), (r_j, [c, d])) = D((r_j, [c, d]), (r_i, [a, b]))$.

4. Since

$$\begin{aligned} D((r_i, [a, b]), (r_k, [e, f])) &= |i - k| + |a - e| + |b - f| \\ &= |i - j + j - k| + |a - c + c - e| + |b - d + d - f| \\ &\leq |i - j| + |j - k| + |a - c| + |c - e| + |b - d| + |d - f| \\ &\leq |i - j| + |a - c| + |b - d| + |j - k| + |c - e| + |d - f| \\ &\leq D((r_i, [a, b]), (r_j, [c, d])) + D((r_j, [c, d]), (r_k, [e, f])) \end{aligned}$$

□

4. TOPSIS to multi-criteria group decision making

In general, multi-criteria group decision making problem includes uncertain and imprecise data and information. We consider the multi-criteria group decision making problems where all the criteria values are expressed in IV2T linguistic information, and interactions phenomena among the decision making criteria or preference of decision makers are taken into account. In group decision making problems, aggregation of expert opinions is very important to appropriately perform evaluation process. TOPSIS is proposed for multi-criteria IV2T linguistic group decision making where expert opinions are aggregated by the IV2T weighted averaging (ITWA) operator. TOPSIS is based on the following steps:

- Step 1.** Let $D = \{D_1, D_2, \dots, D_t\}$ be the set of decision makers (DMs) and $v = (v_1, v_2, \dots, v_t)$ be weight vector of DMs, where $v_l \geq 0$, $l = 1, 2, \dots, t$, $\sum_{l=1}^t v_l = 1$. Consider $A = \{A_1, A_2, \dots, A_m\}$ be the set of alternatives, and $B = \{B_1, B_2, \dots, B_n\}$ be the set of attributes/criteria.
- Step 2.** The decision maker D_l evaluates the alternative A_i with respect to the attribute B_j to get $\gamma_{ij}^{(l)}$, then the decision matrices $R^{(l)} = \left(\gamma_{ij}^{(l)}\right)_{m \times n}$ ($l = 1, 2, \dots, t$) are formed.
- Step 3.** Transform the linguistic decision matrix $\tilde{R}^{(l)} = \left(\gamma_{ij}^{(l)}\right)_{m \times n}$ into the IV2T linguistic decision matrix $R^{(l)} = \left(\gamma_{ij}^{(l)}\right)_{m \times n}$. Use the ITWA operator to aggregate all the t decision makers' evaluation values $\left(r_{ij}^{(l)}, [a_{ij}^{(l)}, b_{ij}^{(l)}]\right)$, $l = 1, 2, \dots, t$, to get the collective evaluation values $(r_{ij}, [a_{ij}, b_{ij}])$ ($i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$) as follows.

$$\begin{aligned} \gamma_{ij} &= (r_{ij}, [a_{ij}, b_{ij}]) = ITWA_v(\gamma_{ij}^{(1)}, \gamma_{ij}^{(2)}, \dots, \gamma_{ij}^{(t)}) \\ &= ITWA_v((r_{ij}^{(1)}, [a_{ij}^{(1)}, b_{ij}^{(1)}]), (r_{ij}^{(2)}, [a_{ij}^{(2)}, b_{ij}^{(2)}]), \dots, (r_{ij}^{(t)}, [a_{ij}^{(t)}, b_{ij}^{(t)}])) \\ &= \Delta \left(\sum_{l=1}^n v_l \nabla((r_{ij}^{(l)}, [a_{ij}^{(l)}, b_{ij}^{(l)}])) \right). \end{aligned}$$

- Step 4.** Let Ω_b be a collection of benefit criteria (i.e., the larger C_j , the greater preference) and Ω_c be a collection of cost criteria (i.e., the smaller C_j , the greater preference). The IV2T positive-ideal solution, denoted as $\tilde{A}^+ = (\tilde{V}_1^+ \ \tilde{V}_2^+ \ \dots \ \tilde{V}_n^+)$, and the IV2T negative-ideal solution, denoted as $\tilde{A}^- = (\tilde{V}_1^- \ \tilde{V}_2^- \ \dots \ \tilde{V}_n^-)$, are defined as follows:

$$\tilde{A}^+ = \left[\left(\left(\max_{l=1}^t \left(\max_i \gamma_{ij}^{(l)} \right) \right) \mid j \in \Omega_b, \left(\min_{l=1}^t \left(\min_i \gamma_{ij}^{(l)} \right) \right) \mid j \in \Omega_c \right) \right]_{i=1, 2, \dots, m},$$

$$\tilde{A}^+ = (\tilde{V}_1^+ \ \tilde{V}_2^+ \ \dots \ \tilde{V}_n^+)$$

$$\tilde{A}^- = \left[\left(\left(\min_{l=1}^t \left(\min_i \gamma_{ij}^{(l)} \right) \right) \mid j \in \Omega_b, \left(\max_{l=1}^t \left(\max_i \gamma_{ij}^{(l)} \right) \right) \mid j \in \Omega_c \right) \right]_{i=1, 2, \dots, m},$$

$$\tilde{A}^- = (\tilde{V}_1^- \ \tilde{V}_2^- \ \dots \ \tilde{V}_n^-).$$

Step 5. Construct positive ideal separation matrix (D^+) and negative ideal separation matrix (D^-) which are defined as follows:

$$D^+ = \begin{bmatrix} d(x_{11}, \tilde{V}_1^+) & + & d(x_{12}, \tilde{V}_2^+) & + & \cdots & + & d(x_{1n}, \tilde{V}_n^+) \\ d(x_{21}, \tilde{V}_1^+) & + & d(x_{22}, \tilde{V}_2^+) & + & \cdots & + & d(x_{2n}, \tilde{V}_n^+) \\ \vdots & & \vdots & & \vdots & & \vdots \\ d(x_{m1}, \tilde{V}_1^+) & + & d(x_{m2}, \tilde{V}_2^+) & + & \cdots & + & d(x_{mn}, \tilde{V}_n^+) \end{bmatrix} = \begin{bmatrix} d(A_1, \tilde{A}^+) \\ d(A_2, \tilde{A}^+) \\ \vdots \\ d(A_m, \tilde{A}^+) \end{bmatrix}$$

and

$$D^- = \begin{bmatrix} d(x_{11}, \tilde{V}_1^-) & + & d(x_{12}, \tilde{V}_2^-) & + & \cdots & + & d(x_{1n}, \tilde{V}_n^-) \\ d(x_{21}, \tilde{V}_1^-) & + & d(x_{22}, \tilde{V}_2^-) & + & \cdots & + & d(x_{2n}, \tilde{V}_n^-) \\ \vdots & & \vdots & & \vdots & & \vdots \\ d(x_{m1}, \tilde{V}_1^-) & + & d(x_{m2}, \tilde{V}_2^-) & + & \cdots & + & d(x_{mn}, \tilde{V}_n^-) \end{bmatrix} = \begin{bmatrix} d(A_1, \tilde{A}^-) \\ d(A_2, \tilde{A}^-) \\ \vdots \\ d(A_m, \tilde{A}^-) \end{bmatrix}$$

Step 6. Calculate the relative closeness (RC) of each alternative to the ideal solution as follows:

$$RC(A_i) = \frac{d(A_i, A^-)}{d(A_i, A^+) + d(A_i, A^-)}, \quad i = 1, 2, \dots, m,$$

Step 7. Rank all the alternatives A_i ($i = 1, 2, \dots, m$) according to the closeness coefficient $RC(A_i)$, the greater the value $RC(A_i)$, the better the alternative A_i .

5. An Illustrative example

Here it is assumed that there is an investment company and board of directors are interested to invest money in the best option (adapted from [10]).

Step 1. There are five possible alternatives in which to invest the money: A_1 is a car industry; A_2 is a food company; A_3 is a computer company; A_4 is an arms company; A_5 is a TV company. Suppose that there are three decision makers D_i ($i = 1, 2, 3$) in the committee, whose weight vector is $v = (0.2, 0.5, 0.3)$. Four attributes B_i ($i = 1, 2, 3, 4$) are used to evaluate the alternatives: B_1 –growth analysis; B_2 –risk analysis; B_3 –environmental impact analysis; B_4 –social political impact analysis.

Step 2. The decision makers evaluate the alternatives with respect to the attributes in IV2T arguments to form decision matrices $\tilde{R}^{(l)} = (r_{ij}^{(l)})_{5 \times 4}$ ($l = 1, 2, \dots, t$), as shown in Tables 1-3.

Table 1

The decision matrix $\tilde{R}^{(1)}$.

	B_1	B_2	B_3	B_4
A_1	(M,[4.1,4.3])	(G,[5.1,5.4])	(P,[2.8,3])	(P,[3.1,3.4])
A_2	(P,[2.9,3])	(M,[3.7,4.1])	(M,[3.9,4])	(G,[5.1,5.4])
A_3	(G,[5.1,5.4])	(M,[4.1,4.4])	(VG,[5.8,6])	(P,[3.1,3.4])
A_4	(VG,[5.8,6])	(P,[3,3.1])	(P,[3,3.2])	(M,[4.1,4.4])
A_5	(EG,[7,7.1])	(P,[2.9,3.2])	(VP,[2,2.3])	(G,[4.8,5.1])

Table 2

The decision matrix $\tilde{R}^{(2)}$.

	B_1	B_2	B_3	B_4
A_1	(P,[2.8,3.1])	(VG,[5.9,6.1])	(VP,[2.1,2.4])	(M,[4.1,4.3])
A_2	(VP,[1.9,2.1])	(P,[2.8,3.1])	(G,[4.7,4.9])	(VG,[5.1,5.3])
A_3	(M,[3.9,4.1])	(P,[2.9,3])	(G,[5,5.1])	(VP,[1.9,2])
A_4	(EG,[6.9,7.1])	(M,[4.1,4.2])	(P,[2.8,3.1])	(G,[4.8,5.1])
A_5	(G,[4.9,5.1])	(M,[4,4.3])	(P,[3,3.2])	(VG,[5.9,6.1])

Table 3

The decision matrix $\tilde{R}^{(3)}$.

	B_1	B_2	B_3	B_4
A_1	(G,[4.9,5.1])	(VG,[5.8,6.2])	(M,[3.9,4.1])	(VP,[1.9,2.1])
A_2	(M,[3.8,4])	(P,[2.9,3.1])	(VG,[5.8,5.9])	(M,[3.6,3.8])
A_3	(P,[2.9,3.1])	(VG,[6.1,6.3])	(G,[4.7,4.9])	(VP,[1.7,2.2])
A_4	(G,[4.8,5.2])	(G,[4.8,5.1])	(P,[2.8,3.1])	(M,[3.7,3.9])
A_5	(M,[3.9,4.1])	(P,[2.9,3.1])	(M,[4,4.2])	(EG,[6.8,7])

Step 3. Use the ITWA operator to aggregate all the decision matrices $\tilde{R}^{(i)}$ ($i = 1, 2, 3$), into the collective decision matrix $\tilde{R}_{5 \times 4}$ as in Table 4.

Table 4

The decision matrix $\tilde{R}$.

	B_1	B_2	B_3	B_4
A_1	(M,[4,4.05])	(VG,[6,6.08])	(P,[3,3.05])	(P,[3,3.42])
A_2	(P,[2.98,3])	(P,[3,3.49])	(G,[5,5.25])	(G,[4.92,5])
A_3	(M,[4,4.12])	(M,[4,4.27])	(G,[5,5.35])	(VP,[2,2.46])
A_4	(VG,[6,6.46])	(M,[4,4.26])	(P,[3,3.28])	(G,[4.77,5])
A_5	(G,[5,5.28])	(M,[3.77,4])	(P,[3,3.32])	(VG,[6,6.32])

Step 4. For cost criteria B_2, B_3 and benefit criteria B_1, B_4 , the positive and negative ideal solutions are in tables 5 and 6, respectively.

Table 5

The positive ideal solution.

	B_1	B_2	B_3	B_4
$\tilde{A}^+$	(EG,[7,7.1])	(P,[2.8,3])	(VP,[2,2.3])	(EG,[6.8,7])

Table 6

The negative ideal solution.

	B_1	B_2	B_3	B_4
$\tilde{A}^-$	(VP,[1.9,2.1])	(VG,[6.1,6.3])	(VG,[5.8,6])	(VP,[1.7,2])

Step 5. Positive ideal separation matrix:

$$D^+ = \begin{bmatrix} 9.05 & + & 9.28 & + & 2.75 & + & 11.38 \\ 12.12 & + & 0.69 & + & 8.95 & + & 5.88 \\ 8.98 & + & 3.47 & + & 9.05 & + & 14.34 \\ 2.64 & + & 3.46 & + & 2.98 & + & 6.03 \\ 5.82 & + & 2.97 & + & 3.02 & + & 2.48 \end{bmatrix} = \begin{bmatrix} 32.46 \\ 27.64 \\ 35.84 \\ 15.11 \\ 14.29 \end{bmatrix}$$

Negative ideal separation matrix:

$$D^- = \begin{bmatrix} 6.05 & + & 0.32 & + & 8.75 & + & 3.72 \\ 2.98 & + & 8.91 & + & 2.55 & + & 9.22 \\ 6.12 & + & 6.13 & + & 2.45 & + & 0.76 \\ 12.46 & + & 6.14 & + & 8.52 & + & 9.07 \\ 9.28 & + & 6.63 & + & 8.48 & + & 12.62 \end{bmatrix} = \begin{bmatrix} 18.84 \\ 23.66 \\ 15.46 \\ 36.19 \\ 37.01 \end{bmatrix}$$

Step 6. Relative closeness of the alternatives:

$$RC(A_1) = 0.36725, RC(A_2) = 0.4612, RC(A_3) = 0.30136, RC(A_4) = 0.705458 \text{ and } RC(A_5) = 0.7214.$$

Step 7. Ranking of the alternatives: $A_3 \prec A_1 \prec A_2 \prec A_4 \prec A_5$.

Therefore, the most desirable alternative is A_5 .

6. Conclusion

It is difficult for decision makers to exactly measure opinion in the domain of fuzzy set theory. Modified TOPSIS method is proposed for solving decision-making problems under the opinion of finite experts and multiple criteria in IV2T linguistic information. The basic concept of the proposed method is based on the positive and negative ideal solutions. The relative closeness coefficient is calculated for each alternative to rank the alternatives by simultaneously taking account of not only the shortest distance from the positive ideal solution but also the farthest distance from the negative ideal solution. In the previous decision making method for IV2T linguistic information was based on aggregation operator of alternatives with respect to criteria [1]. The proposed method is different from this previous technique for group decision making due to the fact that the proposed method used aggregation operator to aggregate the decision of experts and TOPSIS to rank the alternatives. It is efficient for real world decision making problems.

References

- [1] I. Beg and T. Rashid, Aggregation operators of interval-valued 2-tuple linguistic information, *International Journal of Intelligent Systems*, 29 (2014) 634–667.
- [2] G. Büyükoçkan and D. Ruan, Choquet integral based aggregation approach to software development risk assessment, *Information Sciences*, 180 (2010) 441–451.
- [3] K. H. Chang and T. C. Wen, A novel efficient approach for DFMEA combining 2-tuple and the OWA operator, *Expert Systems with Applications*, 37(3) (2010) 2362–2370.
- [4] G. Choquet, Theory of capacities, *Annales del Institut Fourier*, 5 (1953) 131–295.
- [5] T. Demirel, N. C. Demirel and C. Kahraman, Multi-criteria warehouse location selection using Choquet integral, *Expert Systems with Applications*, 37 (2010) 3943–3952.
- [6] Y. C. Dong, Y. F. Xu, H. Y. Li and B. Feng, The OWA-based consensus operator under linguistic representation models using position indexes, *European Journal of Operational Research*, 203 (2010) 455–463.
- [7] M. Dursun and E. E. Karsak, A fuzzy MCDM approach for personnel selection, *Expert Systems with Applications*, 175(1) (2010) 247–286.
- [8] S. Faizi, T. Rashid and S. Zafar, An outranking method for multi-criteria group decision making using hesitant intuitionistic fuzzy linguistic term sets, *Journal of Intelligent & Fuzzy Systems*, 32 (3) (2017) 2153–2164.
- [9] M. Grabisch and C. Labreuche, A decade of application of the Choquet and Sugeno integrals in multi-criteria decision aid, *Annals of Operation Research*, 175(1) (2010) 247–286.
- [10] F. Herrera, E. Herrera-Viedma and L. Martinez, A fusion approach for managing multi-granularity linguistic term sets in decision making, *Fuzzy Sets and Systems*, 114 (2000) 43–58.
- [11] F. Herrera and L. Martinez, An approach for combining linguistic and numerical information based on the 2-tuple fuzzy linguistic representation model in decision-making, *International Journal of Uncertainty Fuzziness and Knowledge-Based Systems*, 8(5) (2000) 539–562.
- [12] F. Herrera and L. Martinez, A 2-tuple fuzzy linguistic representation model for computing with words, *IEEE Transactions on Fuzzy Systems*, 8 (2000) 746–752.
- [13] Y. C. Hu, Classification performance evaluation of single-layer perceptron with Choquet integral-based TOPSIS, *Applied Intelligence*, 29(3) (2000) 204–215.
- [14] C. L. Hwang and K. Yoon, *Multiple attributes decision making methods and applications*, Berlin, Heidelberg: Springer (1981).

- [15] Y. P. Jiang and Z. P. Fan, Property analysis of the aggregation operators for 2-tuple linguistic information, *Control and Decision*, 18(6) (2003) 754–757.
- [16] L. Martínez, R. M. Rodríguez and F. Herrera, *The 2-tuple Linguistic Model*, Springer (Switzerland), (2015).
- [17] P. Meyer and M. Roubens, On the use of the Choquet integral with fuzzy numbers in multiple criteria decision support, *Fuzzy Sets and Systems*, 157 (2006) 927–938.
- [18] J. M. Moreno, J. M. Morales Des Castillo, C. Porcel and E. Herrera-Viedma, A quality evaluation methodology for health-related websites based on a 2-tuple fuzzy linguistic approach, *Soft Computing*, 14(8) (2005) 887–897.
- [19] R. Nasar, A. Asim and T. Rashid, Mathematical study to demonstrate the utilization of TOPSIS to find the maximum possibility of HCV causes, *Punjab University Journal of Mathematics*, (2020) 52 (4), 65-77.
- [20] Z. Pei and P. Shi, Fuzzy risk analysis based on linguistic aggregation operators. *International Journal of Innovative Computing, Information and Control*, 7(12) (2011) 7105-7117.
- [21] T. Rashid, I. Beg and S. M. Husnine, Robot selection by using generalized interval-valued fuzzy numbers with TOPSIS, *Applied Soft Computing*, 21 (2014) 462-468.
- [22] M. S. Sindhu, T. Rashid and A. Kashif, Modeling of linear programming and extended TOPSIS in decision making problem under the framework of picture fuzzy sets, *PloS one*, 14 (8), e0220957
- [23] P. Wakker, *Additive representations of preferences*, Kluwer Academic Publishers (1999).
- [24] W. P. Wang, A fuzzy linguistic computing approach to supplier evaluation, *Applied Mathematical Modelling*, 34(10) (2010) 3130–3141.
- [25] Z. Wang and G. Klir, *Fuzzy measure theory*, New York: Plenum press (1992).
- [26] G. W. Wei, R. Lin, X. F. Zhao and H. J. Wang, Models for multiple attribute group decision making with 2-tuple linguistic assessment information, *International Journal of Computational Intelligence Systems*, 3(3) (2010) 315–324.
- [27] Z. S. Xu, Correlated linguistic information aggregation, *International Journal of Uncertainty Fuzziness and Knowledge-Based Systems*, 17(5) (2009) 633–647.
- [28] R. R. Yager, Induced aggregation operators, *Fuzzy Sets and Systems*, 137 (2003) 59–69.
- [29] W. Yang and Z. Chen, New aggregation operators based on the Choquet integral and 2-tuple linguistic information, *Expert Systems with Applications*, 39 (2012) 2662–2668.
- [30] X.-Y. You, J.-X. You, H.-C. Liu and L. Zhen, Group multi-criteria supplier selection using an extended VIKOR method with interval 2-tuple linguistic information, *Expert Systems with Applications*, 42(4) (2015) 1906–1916.
- [31] L. A. Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338–356.
- [32] Y. Zhang and Z. P. Fan, An approach to linguistic multiple attribute decision-making with linguistic information based on ELOWA operator, *Systems Engineer*, 24(12) (2006) 98–101.