

Computing Analysis of Irregularity Indices for Rhombus type Silicate and Oxide Networks

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Abstract

Topological index (TI) is a numeric quantity that predicts various physical, chemical, thermodynamical and structural properties of various (molecular) graphs. An irregular index is a TI which is used to measure the irregularity of atoms with respect to their bonding for various chemical structures. The rhombus silicate and rhombus oxide are the classes of minerals which are formed by fusing metal oxide with sand and all the silicates containing SiO_4 tetrahedron. In this paper, we will work out connection based irregularity indices of rhombus type silicate and oxide networks. A comparison among the obtained indices with the help of their numerical values and 3D presentations is also included. Finally, among the obtained irregularity indices the efficient and steady indices of rhombus type silicate and oxide networks are addressed.

Keywords: Topological descriptors; Connection based irregularity indices, Rhombus silicate, Rhombus oxide.

Mathematics Subject Classification: : 05C09, 05C92

1. Introduction

Graph theory is used to study various networks and different chemical compounds. The combination of computer science, scientific working fields of chemistry and information science leads to a new subject known as cheminformatics. It is used to transform data into information and information into knowledge for making better decisions faster in the area of drug lead identification and optimization. It studies different physico-chemical and structural properties of the compounds. These properties in chemical compounds are temperature, weight, density, melting point, boiling point, freezing point, solubility, chemical stability, surface tension, reactivity, toxicity, oxidation states, heat of combustion enthalpy of formation. For this purpose various mathematical tools and models are used in cheminformatics however, topological indices (TIs) are the most suitable (see 1 – 3).

TIs are numerical quantities which characterise the whole graph. There are different types of topological indices such as distance-based, degree-based and polynomial related indices of the graphs. In 1947, Wiener calculated the boiling point of a molecular network called paraffin (see 4). Further, in 1972,

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Gutman and Trinajsti worked on degree based TIs to calculate exact total energy of π -electron of different conjugated molecules in [5].

Classification of silicates itself incorporates more than 500 unique minerals. Silicates are one of the most abundant mineral and 87% of the earth's crust is made up of silicates. Each silicon atom comprises the centre of a tetrahedron. According to octet rule an atom or element must have eight electrons in the valence shell. So corners occupied by oxygen atoms are associated with single covalent bond. Silicates are acquired by combining metal oxides or metal carbonates from sand.

Silicates and silicate-based molecules are utilized materials in dentistry. One of their significant application is their utilization as fillers in various dental filling materials such as adhesive systems and glass-ionomer cements. In these materials, the fillers proceed with acids and improved the mechanical properties by increasing radiopacity, thermal expansion coefficient and physical resistance in acrylic filling materials.

Silicates are acquired by combining metal oxides or metal carbonates from sand. Silicate networks are built by different silicate structures as orthosilicates contain discrete SiO_4 units. Pyrosilicates are obtained when two SiO_4 tetrahedrons share an oxygen node. Chain silicates are obtained when tetrahedrons are arranged linearly. Some other silicates oxide networks are dominating silicate network (DSL), dominating oxide network (DOX), rhombus silicate (RHSL), rhombus oxide (RHOX). Rhombus silicate and oxide are formed by fusing metal oxide with sand and all the silicates contain SiO_4 tetrahedron as it is a basic unit.

Farahani *et.al* computed degree based topological indices for chemical graphs of copper oxide type networks in [6]. Simonraj and George calculated topological properties of few Poly Oxide, Poly Silicate, dominating oxide network (DOX) and dominating silicate network (DSL) networks (see 7). Baig *et.al* calculated different TIs for dominating silicate (DSL(n)) and dominating oxide (DOX(n)) networks (see 8). Javaid *et.al* calculated topological indices of rhombus type silicate and oxide networks (see 9). Rajan *et.al* calculated Randic index, Zagreb and atom-bond connectivity indices of honeycomb, silicate and hexagonal networks in [19]. Mondal *et.al* calculated some new neighborhood degree-based indices for some oxide and silicate networks (see 26).

Gutman calculated irregularity of quasi-perfect molecular graphs in [10, 15]. Gao *et.al* calculated irregularity of different dendrimers structures (see 20, 21). Liu *et.al* calculated size of maximally irregular graphs and maximally irregular triangle-free graphs in [12]. Abdo *et.al* investigated the change of the total irregularity of graphs with different subdivision operations (see 13 – 18). Zahid *et.al* calculated irregularity measures of some nanotubes, (see 20). Majcher *et.al* discussed highly irregular graphs with extreme number of edges in [11]. Yang *et.al* computational analysis of irregularity indices for boron nanotubes, (see 22). Dimitrov *et.al* discussed graphs with equal irregularity indices (see 23). The irregularity indices have been concentrated as of late in a novel manner (see 24, 25). Albertson presented basic irregularity index in [26].

2. Notations and Preliminaries

Let $G = (V(G), E(G))$ be a simple connected (molecular) graph. In molecular graph atoms and bonds represent vertices and edges respectively. Let $u \in V(G)$ then its degree is denoted by $d(u)$ and defined as the number of vertices attached to u . The connection number of a vertex $u \in V(G)$ is defined as $\tau(u) = \{v \in V(G) : d(u, v) = 2\}$.

Rhombus silicate and rhombus oxide networks are very interesting class of minerals. These are formed by fusing metal oxide with sand and all the silicates contain SiO_4 tetrahedra as it is a basic unit. Molecular

structures of these oxides having corner vertices of SiO_4 tetrahedron represent oxygen nodes and the central vertex represents the silicon nodes. Rhombus silicate network is denoted by $RHSL$, $|V(RHSL)| = 5n^2 + 2n$ and $|E(RHSL)| = 12n^2$ are number of vertices and edges respectively for n dimensional rhombus silicate network. If we remove all silicon atoms from rhombus silicate then we obtained rhombus oxide network is denoted by $RHSL$, $|V(RHOX)| = 3n^2 + 2n$ and $|E(RHOX)| = 6n^2$ are number of vertices and edges respectively for n dimensional rhombus oxide network.

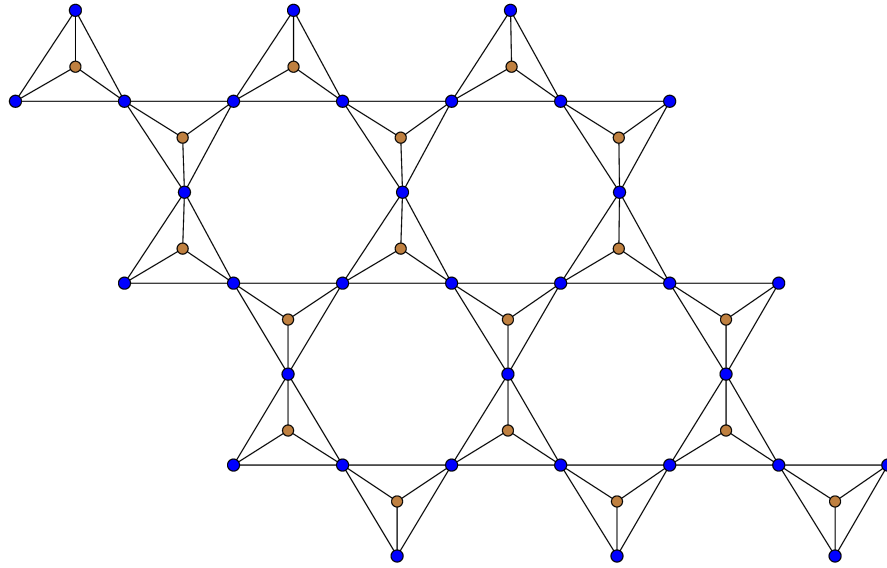


Figure 2.1: Rhombus silicate $RHSL(3)$

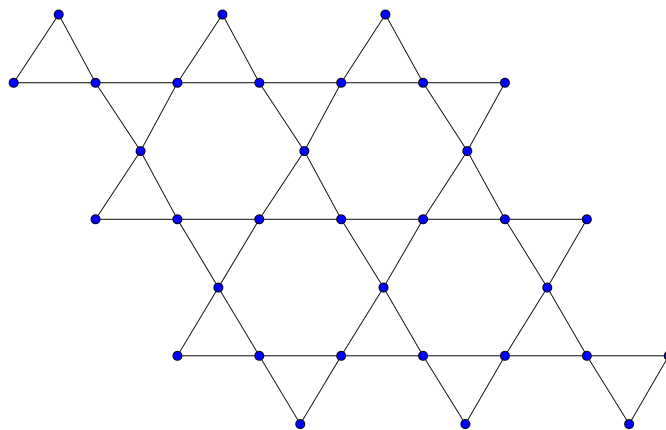


Figure 2.2: Rhombus oxide $RHOX(3)$

Table 1: Connection based irregularity indices's table is given below

Irregularity Index	(Mathematical form
$imball_{uv}^c$	$ \tau_u - \tau_v $
$AL^c(G)$	$\sum_{n_1 n_2 \in E(G)} \tau(n_1) - \tau(n_2) $
$IRL^c(G)$	$\sum_{n_1 n_2 \in E(G)} \ln \tau(n_1) - \ln \tau(n_2) $
$IRR_t^c(G)$	$\frac{1}{2} \sum_{n_1 n_2 \in E(G)} \tau_G(n_1) - \tau_G(n_2) $
$IRF^c(G)$	$\sum_{n_1 n_2 \in E(G)} (\tau_G(n_1) - \tau_G(n_2))^2$
$IRA^c(G)$	$\sum_{n_1 n_2 \in E(G)} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2$
$IRDIF^c(G)$	$\sum_{n_1 n_2 \in E(G)} \left(\left \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_1)} \right \right)$
$LA^c(G)$	$2 \sum_{n_1 n_2 \in E(G)} \frac{ \tau_G(n_1) - \tau_G(n_2) }{\tau(n_1) + \tau(n_2)}$
$IRLF^c(G)$	$\sum_{n_1 n_2 \in E(G)} \frac{ \tau_G(n_1) - \tau_G(n_2) }{\tau(n_1) \tau(n_2)}$
$IRLF^c(G)$	$\sum_{n_1 n_2 \in E(G)} \frac{ \tau_G(n_1) - \tau_G(n_2) }{\sqrt{(\tau(n_2) \tau(n_1))}}$
$IRD1^c(G)$	$\sum_{n_1 n_2 \in E(G)} \ln 1 + \tau_{n_1} + \tau_{n_2} $
$IRGA^c(G)$	$\sum_{n_1 n_2 \in E(G)} \ln \frac{\tau_{n_1} + \tau_{n_2}}{2(\sqrt{\tau_{n_1} \tau_{n_2}})}$
$IRLU^c(G)$	$\frac{\sum_{n_1 n_2 \in E(G)} \ln \tau(n_1) - \ln \tau(n_2) }{\min(\tau(n_1), \tau(n_2))}$
$IRB^c(G)$	$\sum_{n_1 n_2 \in E(G)} (\sqrt{d_{n_1}} - \sqrt{d_{n_2}})^2$

3. Main Results

In this section we will discuss main results.

3.1. Rhombus Oxid

In this section we work out irregularity indices for Rhombus silicate $G \cong RHSL$.

$d(n_1), d(n_1)$	$E_{3,3}^c$	$E_{3,6}^c$	$E_{6,6}^c$	$E_{6,9}^c$	$E_{9,9}^c$	$E_{9,12}^c$	$E_{12,12}^c$
No. of edges	6	6	$4n + 4$	$16n - 14$	$16n - 26$	$6n^2 - 12n$	$6(n - 2)^2$

Table 2: Separation of edge set of edges of RHSL

Theorem 3.1 Let $G \cong RHSL$ be a Rhombus silicate then its connection based irregularity indices are given as

- (i) $IRR^c(G) = 18n^2 + 28n - 24$.
- (ii) $IRL^c(G) = (1.1346) + (16n - 14)(1.3357) + (6n^2 - 12n)(1.8741)$.
- (iii) $IRR_t^c(G) = 9n^2 + 14n - 12$.
- (iv) $IRF^c(G) = 18[3n^2 + 2n - 4]$.

$$(v) \text{IRD1}^c(G) = 2(3n^2 + 2n - 4)(\ln(4)).$$

$$(vi) \text{IRB}^c(G) = (3.0882) + (16n - 14)(0.3030) + (6n^2 - 12n)(0.2153).$$

Proof. Using Table (1)

$$(i) \text{IRR}^c(G) = \sum_{n_1 n_2 \in E^c(G)} |\tau_G(n_1) - \tau_G(n_2)|$$

$$\begin{aligned} &= \sum_{n_1 n_2 \in E_{3,3}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{3,6}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{6,6}^c} |\tau_G(n_1) - \tau_G(n_2)| \\ &+ \sum_{n_1 n_2 \in E_{6,9}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{9,9}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{9,12}^c} |\tau_G(n_1) - \tau_G(n_2)| \\ &+ \sum_{n_1 n_2 \in E_{12,12}^c} |\tau_G(n_1) - \tau_G(n_2)| \\ &= 6|3 - 3| + 6|3 - 6| + (4n + 4)|6 - 6| + (16n - 14)|6 - 9| + (16n - 26)|9 - 9| \\ &+ (6n^2 - 12n)|9 - 12| + (6(n - 2)^2)|12 - 12| \\ &= 6|3 - 6| + (16n - 14)|6 - 9| + (6n^2 - 12n)|9 - 12| \\ &= 18n^2 + 28n - 24 \end{aligned}$$

$$(ii) \text{IRL}^c(G) = \sum_{n_1 n_2 \in E^c(G)} |\ln \tau(n_1) - \ln \tau(n_2)|$$

$$\begin{aligned} &= \sum_{n_1 n_2 \in E_{3,3}^c} |\ln \tau(n_1) - \ln \tau(n_2)| + \sum_{n_1 n_2 \in E_{3,6}^c} |\ln \tau(n_1) - \ln \tau(n_2)| + \sum_{n_1 n_2 \in E_{6,6}^c} |\ln \tau(n_1) - \ln \tau(n_2)| \\ &+ \sum_{n_1 n_2 \in E_{6,9}^c} |\ln \tau(n_1) - \ln \tau(n_2)| + \sum_{n_1 n_2 \in E_{9,9}^c} |\ln \tau(n_1) - \ln \tau(n_2)| + \sum_{n_1 n_2 \in E_{9,12}^c} |\ln \tau(n_1) - \ln \tau(n_2)| \\ &+ \sum_{n_1 n_2 \in E_{12,12}^c} |\ln \tau(n_1) - \ln \tau(n_2)| \\ &= 6|\ln(3) - \ln(3)| + 6|\ln(3) - \ln(6)| + (4n + 4)|\ln(6) - \ln(6)| + (16n - 14)|\ln(6) - \ln(9)| \\ &+ (16n - 26)|\ln(9) - \ln(9)| + (6n^2 - 12n)|\ln(9) - \ln(12)| + (6(n - 2)^2)|\ln(12) - \ln(12)| \\ &= 6|\ln(3) - \ln(6)| + (16n - 14)|\ln(6) - \ln(9)| + (6n^2 - 12n)|\ln(9) - \ln(12)| \\ &= (1.1346) + (16n - 14)(1.3357) + (6n^2 - 12n)(1.8741) \end{aligned}$$

$$(iii) \text{IRR}_t^c(G) = \frac{1}{2} \sum_{n_1 n_2 \in E^c(G)} |\tau_G(n_1) - \tau_G(n_2)|$$

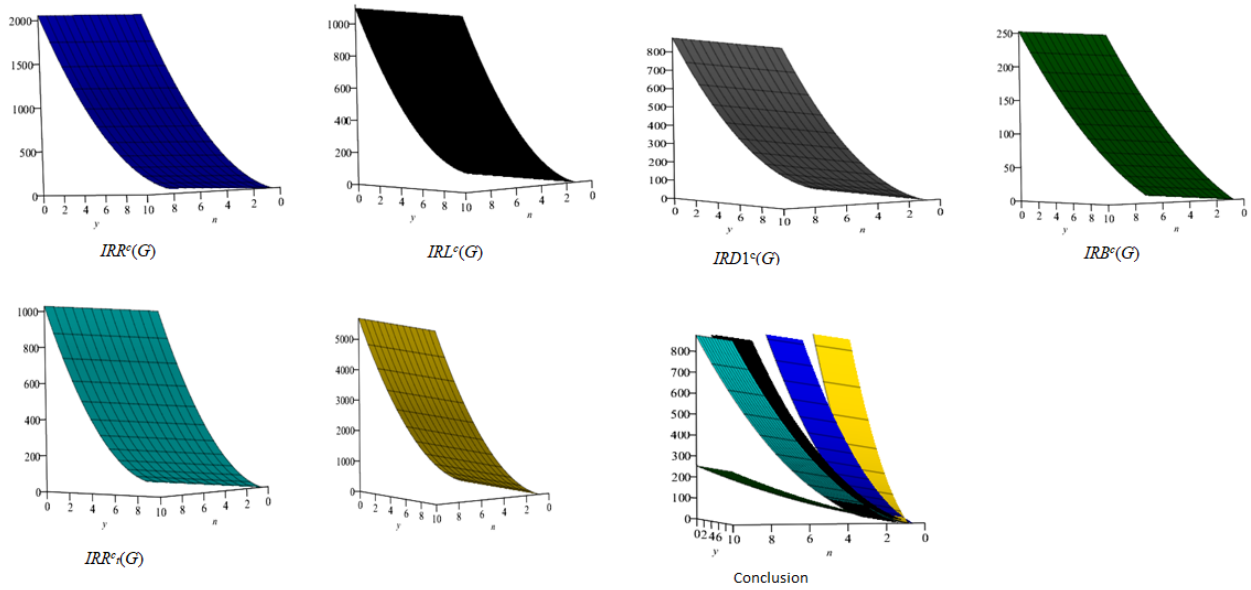
$$\begin{aligned} &= \frac{1}{2} \left[\sum_{n_1 n_2 \in E_{3,3}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{3,6}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{6,6}^c} |\tau_G(n_1) - \tau_G(n_2)| \right. \\ &+ \sum_{n_1 n_2 \in E_{6,9}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{9,9}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{9,12}^c} |\tau_G(n_1) - \tau_G(n_2)| \\ &+ \left. \sum_{n_1 n_2 \in E_{12,12}^c} |\tau_G(n_1) - \tau_G(n_2)| \right] \\ &= \frac{1}{2} [6|3 - 3| + 6|3 - 6| + (4n + 4)|6 - 6| + (16n - 14)|6 - 9| + (16n - 26)|9 - 9| \\ &+ (6n^2 - 12n)|9 - 12| + (6(n - 2)^2)|12 - 12|] \\ &= \frac{1}{2} [6|3 - 6| + (16n - 14)|6 - 9| + (6n^2 - 12n)|9 - 12|] \\ &= 9n^2 + 14n - 12 \end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad IRF^c(G) &= \sum_{n_1 n_2 \in E^c(G)} (\tau_G(n_1) - \tau_G(n_2))^2 \\
&= \sum_{n_1 n_2 \in E_{3,3}^c} (\tau_G(n_1) - \tau_G(n_2))^2 + \sum_{n_1 n_2 \in E_{3,6}^c} (\tau_G(n_1) - \tau_G(n_2))^2 + \sum_{n_1 n_2 \in E_{6,6}^c} (\tau_G(n_1) - \tau_G(n_2))^2 \\
&+ \sum_{n_1 n_2 \in E_{6,9}^c} (\tau_G(n_1) - \tau_G(n_2))^2 + \sum_{n_1 n_2 \in E_{9,9}^c} (\tau_G(n_1) - \tau_G(n_2))^2 + \sum_{n_1 n_2 \in E_{9,12}^c} (\tau_G(n_1) - \tau_G(n_2))^2 \\
&+ \sum_{n_1 n_2 \in E_{12,12}^c} (\tau_G(n_1) - \tau_G(n_2))^2 \\
&= 6(3-3)^2 + 6(3-6)^2 + (4n+4)(6-6)^2 + (16n-14)(6-9)^2 + (16n-26)(9-9)^2 \\
&+ (6n^2-12n)(9-12)^2 + (6(n-2)^2)(12-12)^2 \\
&= 6(3-6)^2 + (16n-14)(6-9)^2 + (6n^2-12n)(9-12)^2 \\
&= 18[3n^2 + 2n - 4]
\end{aligned}$$

$$\begin{aligned}
\text{(v)} \quad IRD1^c(G) &= \sum_{n_1 n_2 \in E^c(G)} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) \\
&= \sum_{n_1 n_2 \in E_{3,3}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) + \sum_{n_1 n_2 \in E_{3,6}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) \\
&+ \sum_{n_1 n_2 \in E_{6,6}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) + \sum_{n_1 n_2 \in E_{6,9}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) \\
&+ \sum_{n_1 n_2 \in E_{9,9}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) + \sum_{n_1 n_2 \in E_{9,12}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) \\
&+ \sum_{n_1 n_2 \in E_{12,12}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) \\
&= 6(\ln(1 + |3-3|)) + 6(\ln(1 + |3-6|)) + (4n+4)(\ln(1 + |6-6|)) + (16n-14)(\ln(1 + |6-9|)) \\
&+ (16n-26)(\ln(1 + |9-9|)) + (6n^2-12n)(\ln(1 + |9-12|)) + (6(n-2)^2)(\ln(1 + |12-12|)) \\
&= 6(\ln(1 + |3-6|)) + (16n-14)(\ln(1 + |6-9|)) + (6n^2-12n)(\ln(1 + |9-12|)) \\
&= 6(\ln(4)) + (16n-14)(\ln(4)) + (6n^2-12n)(\ln(4)) \\
&= 2(3n^2 + 2n - 4)(\ln(4))
\end{aligned}$$

$$\begin{aligned}
\text{(vi)} \quad IRB^c(G) &= \sum_{n_1 n_2 \in E^c(G)} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 \\
&= \sum_{n_1 n_2 \in E_{3,3}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 + \sum_{n_1 n_2 \in E_{3,6}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 + \sum_{n_1 n_2 \in E_{6,6}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 \\
&+ \sum_{n_1 n_2 \in E_{6,9}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 + \sum_{n_1 n_2 \in E_{9,9}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 + \sum_{n_1 n_2 \in E_{9,12}^c} |\tau_G(n_1) - \tau_G(n_2)|(\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 \\
&+ \sum_{n_1 n_2 \in E_{12,12}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 \\
&= 6(\sqrt{3} - \sqrt{3})^2 + 6(\sqrt{3} - \sqrt{6})^2 + (4n+4)(\sqrt{6} - \sqrt{6})^2 + (16n-14)(\sqrt{6} - \sqrt{9})^2 + (16n-26)(\sqrt{9} - \sqrt{9})^2 \\
&+ (6n^2-12n)(\sqrt{9} - \sqrt{12})^2 + (6(n-2)^2)(\sqrt{12} - \sqrt{12})^2 \\
&= 6(\sqrt{3} - \sqrt{6})^2 + (16n-14)(\sqrt{6} - \sqrt{9})^2 + (6n^2-12n)(\sqrt{9} - \sqrt{12})^2 \\
&= (3.0882) + (16n-14)(0.3030) + (6n^2-12n)(0.2153)
\end{aligned}$$

n	$IRR^c(G)$	$IRL^c(G)$	$IRR_t^c(G)$	$IRF^c(G)$	$IRD1^c(G)$	$IRB^c(G)$
(3)	222	80.2822	111	522	80.4050	17.2656
(4)	376	17.5876	188	9366	144.1476	28.5726
(5)	566	257.9598	283	1458	224.5796	42.4630
(6)	792	380.5324	396	2088	321.6202	58.9373
(7)	1054	525.594	427	2826	435.294	77.9952
(8)	1352	693.1452	676	3672	565.6080	99.6366
(9)	1686	883.1854	843	4626	172.5553	123.8616
(10)	2056	1095.7148	1028	5688	876.1380	150.6702
(11)	2464	1330.7334	1232	6858	1056.	180.6024
(12)	2904	1588.2414	1452	8136	1253.2101	2122.0382

Table 3: Table for connection based irregularity indices of $RHSL$ 

Comparison The connection based irregularity indices $IRR^c(G)$, $IRL^c(G)$, $IRR_t^c(G)$, $IRF^c(G)$, $IRD1^c(G)$, $IRB^c(G)$ are presented by blue, black, cyan, yellow, gray and green colours respectively. Moreover, $IRF^c(G)$ is dominant connection based irregularity index for rhombus silicate.

Theorem 3.2 Let $G \cong RHSL$ be a Rhombus silicate then its connection based irregularity indices are given as

- (i) $IRLU^c(G) = 2(0.1891) + (16n - 14)(1.3357) + (6n^2 - 12n)(1.8741)$.
- (ii) $IRA^c(G) = (0.1715) + (16n - 14)(0.0056) + (6n^2 - 12n)(0.0019)$.
- (iii) $IRDIF^c(G) = \frac{9n^2 + 14n - 10}{6}$.
- (iv) $IRLF^c(G) = \sqrt{18} + \frac{(16n - 14)}{\sqrt{6}} + \frac{3(6n^2 - 12n)}{\sqrt{108}}$.
- (v) $LA^c(G) = \frac{2(30n^2 + 52n - 63)}{35}$.
- (vi) $IRGA^c(G) = (6) \ln \frac{9}{2\sqrt{(18)}} + (16n - 14) \ln \frac{15}{2\sqrt{(54)}} + (6n^2 - 12n) \ln \frac{21}{2\sqrt{(108)}}$.

Proof. Using Table (1)

$$(i) \quad IRLU^c(G) = \frac{\sum_{uv \in E(G)} |\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))}$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{3,3}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} + \sum_{n_1 n_2 \in E_{3,6}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} + \sum_{n_1 n_2 \in E_{6,6}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} \\
&+ \sum_{n_1 n_2 \in E_{6,9}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} + \sum_{n_1 n_2 \in E_{9,9}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} + \sum_{n_1 n_2 \in E_{9,12}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} \\
&+ \sum_{n_1 n_2 \in E_{12,12}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} \\
&= (6) \frac{|\ln(3) - \ln(3)|}{\min(3,3)} + (6) \frac{|\ln(3) - \ln(6)|}{\min(3,6)} + (4n+4) \frac{|\ln(6) - \ln(6)|}{\min(6,6)} + (16n-14) \frac{|\ln(6) - \ln(9)|}{\min(6,9)} \\
&+ (16n-26) \frac{|\ln(9) - \ln(9)|}{\min(9,9)} + (6n^2-12n) \frac{|\ln(9) - \ln(12)|}{\min(9,12)} + (6(n-2)^2) \frac{|\ln(12) - \ln(12)|}{\min(12,12)} \\
&= (6) \frac{|\ln(3) - \ln(6)|}{3} + (16n-14) \frac{|\ln(6) - \ln(9)|}{6} + (6n^2-12n) \frac{|\ln(9) - \ln(12)|}{9} \\
&= 2(0.1891 + (16n-14)(1.3357) + (6n^2-12n)(1.8741))
\end{aligned}$$

$$(ii) \ IRA^c(G) = \sum_{n_1 n_2 \in E(G)} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{3,3}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 + \sum_{n_1 n_2 \in E_{3,6}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 + \sum_{n_1 n_2 \in E_{6,6}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 \\
&+ \sum_{n_1 n_2 \in E_{6,9}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 + \sum_{n_1 n_2 \in E_{9,9}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 + \sum_{n_1 n_2 \in E_{9,12}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 \\
&+ \sum_{n_1 n_2 \in E_{12,12}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 \\
&= (6) \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \right)^2 + (6) \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} \right)^2 + (4n+4) \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}} \right)^2 \\
&+ (16-14) \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{9}} \right)^2 + (16n-26) \left(\frac{1}{\sqrt{9}} - \frac{1}{\sqrt{9}} \right)^2 + (6n^2-12n) \left(\frac{1}{\sqrt{9}} - \frac{1}{\sqrt{12}} \right)^2 \\
&+ ((6n-2)^2) \left(\frac{1}{\sqrt{12}} - \frac{1}{\sqrt{12}} \right)^2 \\
&= (6) \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} \right)^2 + (16-14) \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{9}} \right)^2 + (6n^2-12n) \left(\frac{1}{\sqrt{9}} - \frac{1}{\sqrt{12}} \right)^2 \\
&= (0.1715) + (16n-14)(0.0056) + (6n^2-12n)(0.0019)
\end{aligned}$$

$$(iii) \ IRDIF^c(G) = \sum_{n_1 n_2 \in E(G)} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_1)} \right| \right)$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{3,3}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) + \sum_{n_1 n_2 \in E_{3,6}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) + \sum_{n_1 n_2 \in E_{6,6}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) \\
&+ \sum_{n_1 n_2 \in E_{6,9}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) + \sum_{n_1 n_2 \in E_{9,9}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) + \sum_{n_1 n_2 \in E_{9,12}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) \\
&+ \sum_{n_1 n_2 \in E_{12,12}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) \\
&= (6) \left(\left| \frac{3}{3} - \frac{3}{3} \right| \right) + (6) \left(\left| \frac{3}{6} - \frac{6}{6} \right| \right) + (4n+4n) \left(\left| \frac{6}{6} - \frac{6}{6} \right| \right) + (16n-14) \left(\left| \frac{6}{9} - \frac{9}{9} \right| \right) \\
&+ (16n-26) \left(\left| \frac{9}{9} - \frac{9}{9} \right| \right) + (6n^2-12n) \left(\left| \frac{9}{12} - \frac{12}{12} \right| \right) + ((6n-2)^2) \left(\left| \frac{12}{12} - \frac{12}{12} \right| \right) \\
&= (6) \left(\left| \frac{1}{3} - 1 \right| \right) + (16n-14) \left(\left| \frac{6}{9} - 1 \right| \right) + (6n^2-12n) \left(\left| \frac{9}{12} - 1 \right| \right) = \frac{9n^2+14n-10}{6}
\end{aligned}$$

$$(iv) \ IRLF^c(G) = \sum_{n_1 n_2 \in E(G)} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}}$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{3,3}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{3,6}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{6,6}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} \\
&+ \sum_{n_1 n_2 \in E_{6,9}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{9,9}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{9,12}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} \\
&+ \sum_{n_1 n_2 \in E_{12,12}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} \\
&= (6) \frac{|3-3|}{\sqrt{(3)(3)}} + (6) \frac{|3-6|}{\sqrt{(3)(6)}} + (4n+4) \frac{|6-6|}{\sqrt{(6)(6)}} + (16n-14) \frac{|6-9|}{\sqrt{(6)(9)}} + (16n-26) \frac{|9-9|}{\sqrt{(9)(9)}} \\
&+ (6n^2-12n) \frac{|9-12|}{\sqrt{(9)(12)}} + ((6n-2)^2) \frac{|12-12|}{\sqrt{(12)(12)}} \\
&= (6) \frac{|-3|}{\sqrt{(18)}} + (16n-14) \frac{|-3|}{3\sqrt{(6)}} + (6n^2-12n) \frac{|-3|}{\sqrt{(108)}} \\
&= \sqrt{18} + \frac{(16n-14)}{\sqrt{6}} + \frac{3(6n^2-12n)}{\sqrt{108}}
\end{aligned}$$

$$(v) \ LA^c(G) = 2 \sum_{n_1 n_2 \in E(G)} \frac{|d_G(n_1) - d_G(n_2)|}{d(n_1) + d(n_2)}$$

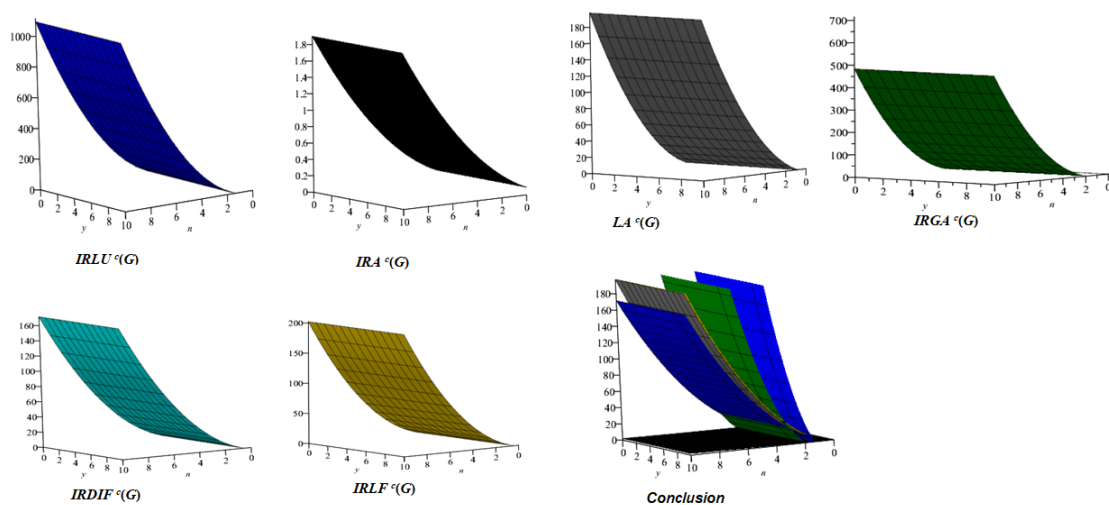
$$\begin{aligned}
&= 2 \left[\sum_{n_1 n_2 \in E_{3,3}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} + \sum_{n_1 n_2 \in E_{3,6}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} + \sum_{n_1 n_2 \in E_{6,6}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} \right. \\
&+ \sum_{n_1 n_2 \in E_{6,9}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} + \sum_{n_1 n_2 \in E_{9,9}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} + \sum_{n_1 n_2 \in E_{9,12}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} \\
&+ \left. \sum_{n_1 n_2 \in E_{12,12}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} \right] \\
&= 2 \left[(6) \frac{|3-3|}{(3)+(3)} + (6) \frac{|3-6|}{(3)+(6)} + (4n+4) \frac{|6-6|}{(6)+(6)} + (16n-14) \frac{|6-9|}{(6)+(9)} + (16n-26) \frac{|9-9|}{(9)+(9)} \right. \\
&+ \left. (6n^2-12n) \frac{|9-12|}{(9)+(12)} + ((6n-2)^2) \frac{|12-12|}{(12)+(12)} \right] \\
&= 2 \left[(6) \frac{|-3|}{(9)} + (16n-14) \frac{|-3|}{15} + (6n^2-12n) \frac{|-3|}{21} \right] \\
&= \frac{2(30n^2 + 52n - 63)}{35}
\end{aligned}$$

$$(\text{vi}) \quad IRGA^c(G) = \sum_{n_1 n_2 \in E(G)} \ln \frac{\tau_{n_1} + \tau_{n_2}}{2(\sqrt{\tau_{n_1} \tau_{n_2}})}$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{3,3}^c} \ln \frac{|\tau_G(n_1) + \tau_G(n_2)|}{2\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{3,6}^c} \ln \frac{|\tau_G(n_1) + \tau_G(n_2)|}{2\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{6,6}^c} \ln \frac{|\tau_G(n_1) + \tau_G(n_2)|}{2\sqrt{(\tau(n_1)\tau(n_2))}} \\
&+ \sum_{n_1 n_2 \in E_{6,9}^c} \ln \frac{|\tau_G(n_1) + \tau_G(n_2)|}{2\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{9,9}^c} \ln \frac{|\tau_G(n_1) + \tau_G(n_2)|}{2\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{9,12}^c} \ln \frac{|\tau_G(n_1) + \tau_G(n_2)|}{2\sqrt{(\tau(n_1)\tau(n_2))}} \\
&+ \sum_{n_1 n_2 \in E_{12,12}^c} \ln \frac{|\tau_G(n_1) + \tau_G(n_2)|}{2\sqrt{(\tau(n_1)\tau(n_2))}} \\
&= (6) \ln \frac{|3+3|}{2\sqrt{(3)(3)}} + (6) \ln \frac{|3+6|}{2\sqrt{(3)(6)}} + (4n+4) \ln \frac{|6+6|}{2\sqrt{(6)(6)}} + (16n-14) \ln \frac{|6+9|}{2\sqrt{(6)(9)}} \\
&+ (16n-26) \ln \frac{|9+9|}{2\sqrt{(9)(9)}} + (6n^2-12n) \ln \frac{|9+12|}{2\sqrt{(9)(12)}} + ((6n-2)^2) \ln \frac{|12+12|}{2\sqrt{(12)(12)}} \\
&= (6) \ln \frac{9}{2\sqrt{(18)}} + (16n-14) \ln \frac{15}{2\sqrt{(54)}} + (6n^2-12n) \ln \frac{21}{2\sqrt{(108)}}
\end{aligned}$$

Comparison The connection based irregularity indices $IRLU^c(G)$, $IRA^c(G)$, $IRDIF^c(G)$, $IRLF^c(G)$, $LA^c(G)$, $IRGA^c(G)$ presented by blue, black, cyan, yellow, gray and green colours. Moreover, $IRGA^c(G)$ is dominant connection based irregularity index for rhombus silicate.

n	$IRLU^c(G)$	$IRA^c(G)$	$IRDIF^c(G)$	$IRLF^c(G)$	$LA^c(G)$	$IRGA^c(G)$
(3)	79.5258	0.3961	18.833	23.3192	20.7428	179.6964
(4)	157.12	0.5427	31.666	38.5114	39.7142	351.5727
(5)	257.2034	0.5121	47.5	57.1677	54.1142	549.4490
(6)	379.776	0.9034	66.333	79.2888	75.9428	791.3253
(7)	524.8378	1.1193	88.1166	104.8727	101.2	1072
(8)	692.3888	1.3571	113	133.9213	129.8857	1407.0778
(9)	882.429	1.6177	140.8383	166.4341	162	1780.9541
(10)	1094.9584	1.9011	171.666	202.4109	197.5428	2198.8304
(11)	1329.977	2.2073	205.5	241.8518	236.5442	2660.7067
(12)	1587.484	2.5663	243.333	284.7569	278.9142	3166.5830

Table 4: Table for connection based irregularity indices of $RHSL$ 

3.2. Rhombus Oxid

In this section we work out irregularity indices for Rhombus oxid $G \cong RHOX$.

$d(n_1), d(n_1)$	$E_{2,2}^c$	$E_{2,4}^c$	$E_{4,4}^c$	$E_{4,6}^c$	$E_{6,6}^c$	$E_{6,8}^c$	$E_{8,8}^c$
No. of edges	2	4	4	$8n - 4$	$8n - 14$	$8n - 16$	$6n^2 - 24(n - 1)$

Table 5: Separation of edge set of $RHOX$.

Theorem 4.1 Let $G \cong RHOX$ be a Rhombus oxid then its connection based irregularity indices are given as

- (i) $IRR^c(G) = 32(n - 1)$.
- (ii) $IRL^c(G) = (1.9528) + (8n - 4)(0.7921) + (8n - 16)(1.3662)$.
- (iii) $IRR_t^c(G) = 16(n - 1)$.
- (iv) $IRF^c(G) = 64(n - 1)$.
- (v) $IRD1^c(G) = 16(n - 1)(\ln(3))$.
- (vi) $IRB^c(G) = (1.3724) + (8n - 4)(0.2020) + (8n - 16)(0.1435)$.

Proof. Using Table (1)

$$(i) \quad IRR^c(G) = \sum_{n_1 n_2 \in E^c(G)} |\tau_G(n_1) - \tau_G(n_2)|$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{2,2}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{2,4}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{4,4}^c} |\tau_G(n_1) - \tau_G(n_2)| \\
&+ \sum_{n_1 n_2 \in E_{4,6}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{6,6}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{6,8}^c} |\tau_G(n_1) - \tau_G(n_2)| \\
&+ \sum_{n_1 n_2 \in E_{8,8}^c} |\tau_G(n_1) - \tau_G(n_2)| \\
&= 2|2-2| + 4|2-4| + 4|4-4| + (8n-4)|4-6| + (8n-14)|6-6| \\
&+ (8n-16)|6-8| + (6n^2-24(n-1))|8-8| \\
&= 4|-2| + (8n-4)|-2| + (8n-16)|-2| \\
&= 32(n-1)
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad IRL^c(G) &= \sum_{n_1 n_2 \in E^c(G)} |\ln \tau(n_1) - \ln \tau(n_2)| \\
&= \sum_{n_1 n_2 \in E_{2,2}^c} |\ln \tau(n_1) - \ln \tau(n_2)| + \sum_{n_1 n_2 \in E_{2,4}^c} |\ln \tau(n_1) - \ln \tau(n_2)| + \sum_{n_1 n_2 \in E_{4,4}^c} |\ln \tau(n_1) - \ln \tau(n_2)| \\
&+ \sum_{n_1 n_2 \in E_{4,6}^c} |\ln \tau(n_1) - \ln \tau(n_2)| + \sum_{n_1 n_2 \in E_{6,6}^c} |\ln \tau(n_1) - \ln \tau(n_2)| + \sum_{n_1 n_2 \in E_{6,8}^c} |\ln \tau(n_1) - \ln \tau(n_2)| \\
&+ \sum_{n_1 n_2 \in E_{8,8}^c} |\ln \tau(n_1) - \ln \tau(n_2)| \\
&= 2|\ln(2) - \ln(2)| + 4|\ln(2) - \ln(4)| + 4|\ln(4) - \ln(4)| + (8n-4)|\ln(4) - \ln(6)| \\
&+ (8n-14)|\ln(6) - \ln(6)| + (8n-16)|\ln(6) - \ln(8)| + (6n^2-24(n-1))|\ln(8) - \ln(8)| \\
&= 4(0.4882) + (8n-4)(0.7921) + (8n-16)(1.3662) \\
&= (1.9528) + (8n-4)(0.7921) + (8n-16)(1.3662)
\end{aligned}$$

$$\begin{aligned}
\text{(iii)} \quad IRR_t^c(G) &= \frac{1}{2} \sum_{n_1 n_2 \in E^c(G)} |\tau_G(n_1) - \tau_G(n_2)| \\
&= \frac{1}{2} \left[\sum_{n_1 n_2 \in E_{2,2}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{2,4}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{4,4}^c} |\tau_G(n_1) - \tau_G(n_2)| \right. \\
&+ \sum_{n_1 n_2 \in E_{4,6}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{6,6}^c} |\tau_G(n_1) - \tau_G(n_2)| + \sum_{n_1 n_2 \in E_{6,8}^c} |\tau_G(n_1) - \tau_G(n_2)| \\
&+ \left. \sum_{n_1 n_2 \in E_{8,8}^c} |\tau_G(n_1) - \tau_G(n_2)| \right] \\
&= \frac{1}{2} [2|2-2| + 4|2-4| + 4|4-4| + (8n-4)|4-6| + (8n-14)|6-6| \\
&+ (8n-16)|6-8| + (6n^2-24(n-1))|8-8|] \\
&= \frac{1}{2} [4|-2| + (8n-4)|-2| + (8n-16)|-2|] \\
&= 16(n-1)
\end{aligned}$$

$$\text{(iv)} \quad IRF^c(G) = \sum_{n_1 n_2 \in E^c(G)} (\tau_G(n_1) - \tau_G(n_2))^2$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{2,2}^c} (\tau_G(n_1) - \tau_G(n_2))^2 + \sum_{n_1 n_2 \in E_{2,4}^c} (\tau_G(n_1) - \tau_G(n_2))^2 + \sum_{n_1 n_2 \in E_{4,4}^c} (\tau_G(n_1) - \tau_G(n_2))^2 \\
&+ \sum_{n_1 n_2 \in E_{4,6}^c} (\tau_G(n_1) - \tau_G(n_2))^2 + \sum_{n_1 n_2 \in E_{6,6}^c} (\tau_G(n_1) - \tau_G(n_2))^2 + \sum_{n_1 n_2 \in E_{6,8}^c} (\tau_G(n_1) - \tau_G(n_2))^2 \\
&+ \sum_{n_1 n_2 \in E_{8,8}^c} (\tau_G(n_1) - \tau_G(n_2))^2 \\
&= 2(2-2)^2 + 4(2-4)^2 + 4(4-4)^2 + (8n-4)(4-6)^2 + (8n-14)(6-6)^2 \\
&+ (8n-16)(6-8)^2 + (6n^2-24(n-1))(8-8)^2 \\
&= 4(4) + (8n-4)(4) + (8n-16)(4) \\
&= 64(n-1)
\end{aligned}$$

$$(v) \text{IRD}1^c(G) = \sum_{n_1 n_2 \in E^c(G)} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|))$$

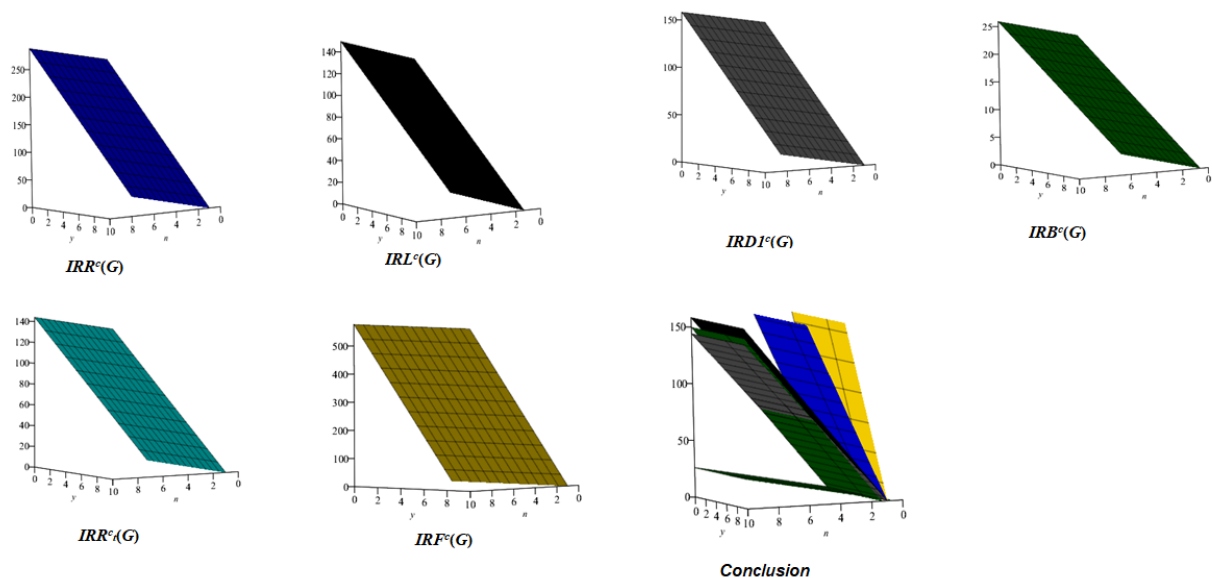
$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{2,2}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) + \sum_{n_1 n_2 \in E_{2,4}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) \\
&+ \sum_{n_1 n_2 \in E_{4,4}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) + \sum_{n_1 n_2 \in E_{4,6}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) \\
&+ \sum_{n_1 n_2 \in E_{6,6}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) + \sum_{n_1 n_2 \in E_{6,8}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) \\
&+ \sum_{n_1 n_2 \in E_{8,8}^c} (\ln(1 + |\tau_G(n_1) + \tau_G(n_2)|)) \\
&= 2(\ln(1 + |2-2|)) + 4(\ln(1 + |2-4|)) + 4(\ln(1 + |4-4|)) + (8n-4)(\ln(1 + |4-6|)) \\
&+ (8n-14)(\ln(1 + |6-6|)) + (8n-16)(\ln(1 + |6-8|)) + (6n^2-24(n-1))(\ln(1 + |8-8|)) \\
&= 4(\ln(3)) + (8n-4)(\ln(3)) + (8n-16)(\ln(3)) \\
&= 16(n-1)(\ln(3))
\end{aligned}$$

$$(vi) \text{IRB}^c(G) = \sum_{n_1 n_2 \in E^c(G)} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{2,2}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 + \sum_{n_1 n_2 \in E_{2,4}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 + \sum_{n_1 n_2 \in E_{4,4}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 \\
&+ \sum_{n_1 n_2 \in E_{4,6}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 + \sum_{n_1 n_2 \in E_{6,6}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 + \sum_{n_1 n_2 \in E_{6,8}^c} |\tau_G(n_1) - \tau_G(n_2)|(\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 \\
&+ \sum_{n_1 n_2 \in E_{8,8}^c} (\sqrt{\tau_{n_1}} - \sqrt{\tau_{n_2}})^2 \\
&= 2(\sqrt{2} - \sqrt{2})^2 + 4(\sqrt{2} - \sqrt{4})^2 + 4(\sqrt{4} - \sqrt{4})^2 + (8n-4)(\sqrt{4} - \sqrt{6})^2 + (8n-14)(\sqrt{6} - \sqrt{6})^2 \\
&+ (8n-16)(\sqrt{6} - \sqrt{8})^2 + (6n^2-24(n-1))(\sqrt{8} - \sqrt{8})^2 \\
&= (1.3724) + (8n-4)(0.2020) + (8n-16)(0.1435)
\end{aligned}$$

n	$IRR^c(G)$	$IRL^c(G)$	$IRR_l^c(G)$	$IRF^c(G)$	$IRD1^c(G)$	$IRB^c(G)$
(3)	64	28.7244	32	128	35.1555	6.5604
(4)	96	45.9908	48	192	52.7333	9.3244
(5)	128	63.2572	64	256	70.3111	12.0884
(6)	160	80.5236	80	320	87.8889	14.8524
(7)	192	97.79	96	384	105.4667	17.6164
(8)	244	115.0564	122	448	123.0445	20.3804
(9)	256	132.3228	128	512	140.6223	23.1444
(10)	288	149.5892	144	576	158.2001	25.9084
(11)	320	166.8556	160	640	175.7779	28.6724
(12)	352	184.122	176	704	193.3557	31.4364

Table 6: Table for connection based irregularity indices of $RHOX$



Comparison The connection based irregularity indices $IRR^c(G)$, $IRL^c(G)$, $IRR_l^c(G)$, $IRF^c(G)$, $IRD1^c(G)$, $IRB^c(G)$ are presented by blue, black, cyan, yellow, gray and green colours respectively. Moreover, $IRF^c(G)$ is dominant connection based irregularity index for rhombus oxid.

Theorem 4.2 Let $G \cong RHOX$ be a Rhombus oxid then its connection based irregularity indices are given as

- (i) $IRLU^c(G) = (0.9764) + (2n-1)(0.7921) + (8n-16)(0.2277)$.
- (ii) $IRA^c(G) = (0.1712) + (8n-4)(0.0084) + (8n-16)(0.002)$.
- (iii) $IRDIF^c(G) = \frac{14n-10}{3}$.
- (iv) $IRLF^c(G) = 2\sqrt{2} + (\frac{8n-4}{\sqrt{6}}) + \frac{2(8n-16)}{\sqrt{48}}$.
- (v) $LA(G) = (2)(\frac{288n-264}{105})$.
- (vi) $IRGA(G) = 4(\ln\frac{3}{2\sqrt{3}}) + (8n-4)(\ln\frac{5}{\sqrt{24}}) + (8n-16)(\ln\frac{7}{\sqrt{48}})$.

Proof. Using Table (1)

$$\begin{aligned}
 \text{(i) } IRLU^c(G) &= \frac{\sum_{uv \in E(G)} |\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} \\
 &= \sum_{n_1 n_2 \in E_{2,2}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} + \sum_{n_1 n_2 \in E_{2,4}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} + \sum_{n_1 n_2 \in E_{4,4}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} \\
 &+ \sum_{n_1 n_2 \in E_{4,6}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} + \sum_{n_1 n_2 \in E_{6,6}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} + \sum_{n_1 n_2 \in E_{6,8}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} \\
 &+ \sum_{n_1 n_2 \in E_{8,8}^c} \frac{|\ln \tau_G(n_1) - \ln \tau_G(n_2)|}{\min(\tau(n_1), \tau(n_2))} \\
 &= (2) \frac{|\ln(2) - \ln(2)|}{\min(2,2)} + (4) \frac{|\ln(2) - \ln(4)|}{\min(2,4)} + (4) \frac{|\ln(4) - \ln(4)|}{\min(4,4)} + (8n-4) \frac{|\ln(4) - \ln(6)|}{\min(4,6)} \\
 &+ (8n-14) \frac{|\ln(6) - \ln(6)|}{\min(6,6)} + (8n-16) \frac{|\ln(6) - \ln(8)|}{\min(6,8)} + (6n^2 - 24(n-1)) \frac{|\ln(8) - \ln(8)|}{\min(8,8)} \\
 &= (4) \frac{|\ln(2) - \ln(4)|}{\min(2,4)} + (8n-4) \frac{|\ln(4) - \ln(6)|}{\min(4,6)} + (8n-16) \frac{|\ln(6) - \ln(8)|}{\min(6,8)} \\
 &= (4) \frac{|\ln(2) - \ln(4)|}{2} + (8n-4) \frac{|\ln(4) - \ln(6)|}{4} + (8n-16) \frac{|\ln(6) - \ln(8)|}{6} \\
 &= (0.9764) + (2n-1)(0.7921) + (8n-16)(0.2277)
 \end{aligned}$$

$$\text{(iii) } IRA^c(G) = \sum_{n_1 n_2 \in E(G)} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{2,2}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 + \sum_{n_1 n_2 \in E_{2,4}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 + \sum_{n_1 n_2 \in E_{4,4}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 \\
&+ \sum_{n_1 n_2 \in E_{4,6}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 + \sum_{n_1 n_2 \in E_{6,6}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 + \sum_{n_1 n_2 \in E_{6,8}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 \\
&+ \sum_{n_1 n_2 \in E_{8,8}^c} \left(\frac{1}{\sqrt{\tau(n_1)}} - \frac{1}{\sqrt{\tau(n_2)}} \right)^2 \\
&= (2) \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right)^2 + (4) \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{4}} \right)^2 + (4) \left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{4}} \right)^2 \\
&+ (8n-4) \left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{6}} \right)^2 + (8n-14) \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}} \right)^2 + (8n-16) \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{8}} \right)^2 \\
&+ (6n^2 - 24(n-1)) \left(\frac{1}{\sqrt{8}} - \frac{1}{\sqrt{8}} \right)^2 \\
&= (4) \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{4}} \right)^2 + (8n-4) \left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{6}} \right)^2 + (8n-16) \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{8}} \right)^2 \\
&= (0.1712) + (8n-4)(0.0084) + (8n-16)(0.002)
\end{aligned}$$

$$\text{(iv)} \quad \text{IRDIF}^c(G) = \sum_{n_1 n_2 \in E(G)} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right)$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{2,2}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) + \sum_{n_1 n_2 \in E_{2,4}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) + \sum_{n_1 n_2 \in E_{4,4}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) \\
&+ \sum_{n_1 n_2 \in E_{4,6}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) + \sum_{n_1 n_2 \in E_{6,6}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) + \sum_{n_1 n_2 \in E_{6,8}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) \\
&+ \sum_{n_1 n_2 \in E_{8,8}^c} \left(\left| \frac{\tau(n_1)}{\tau(n_2)} - \frac{\tau(n_2)}{\tau(n_2)} \right| \right) \\
&= (2) \left(\left| \frac{2}{2} - \frac{2}{2} \right| \right) + (4) \left(\left| \frac{2}{4} - \frac{4}{4} \right| \right) + (4) \left(\left| \frac{4}{4} - \frac{4}{4} \right| \right) + (8n-4) \left(\left| \frac{4}{6} - \frac{6}{6} \right| \right) \\
&+ (8n-14) \left(\left| \frac{6}{6} - \frac{6}{6} \right| \right) + (8n-16) \left(\left| \frac{6}{8} - \frac{8}{8} \right| \right) + (6n^2 - 24(n-1)) \left(\left| \frac{8}{8} - \frac{8}{8} \right| \right) \\
&= (4) \left(\left| \frac{1}{2} - 1 \right| \right) + (8n-4) \left(\left| \frac{2}{3} - 1 \right| \right) + (8n-16) \left(\left| \frac{3}{4} - 1 \right| \right) \\
&= \frac{14n-10}{3}
\end{aligned}$$

$$\text{(v)} \quad \text{IRLF}^c(G) = \sum_{n_1 n_2 \in E(G)} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{\tau(n_1)\tau(n_2)}}$$

$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{2,2}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{2,4}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{4,4}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} \\
&+ \sum_{n_1 n_2 \in E_{4,6}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{6,6}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} + \sum_{n_1 n_2 \in E_{6,8}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} \\
&+ \sum_{n_1 n_2 \in E_{8,8}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{\sqrt{(\tau(n_1)\tau(n_2))}} \\
&= (2) \frac{|2-2|}{\sqrt{(2)(2)}} + (4) \frac{|2-4|}{\sqrt{(2)(4)}} + (4) \frac{|4-4|}{\sqrt{(4)(4)}} + (8n-4) \frac{|4-6|}{\sqrt{(4)(6)}} + (8n-14) \frac{|6-6|}{\sqrt{(6)(6)}} \\
&+ (8n-16) \frac{|6-8|}{\sqrt{(6)(8)}} + (6n^2 - 24(n-1)) \frac{|8-8|}{\sqrt{(8)(8)}} \\
&= (4) \frac{|-2|}{\sqrt{(2)(4)}} + (8n-4) \frac{|-2|}{\sqrt{(4)(6)}} + (8n-16) \frac{|-2|}{\sqrt{(6)(8)}} \\
&= 2\sqrt{2} + \left(\frac{8n-4}{\sqrt{6}}\right) + \frac{2(8n-16)}{\sqrt{48}}
\end{aligned}$$

$$(vi) \quad LA^c(G) = 2 \sum_{n_1 n_2 \in E(G)} \frac{|d_G(n_1) - d_G(n_2)|}{d(n_1) + d(n_2)}$$

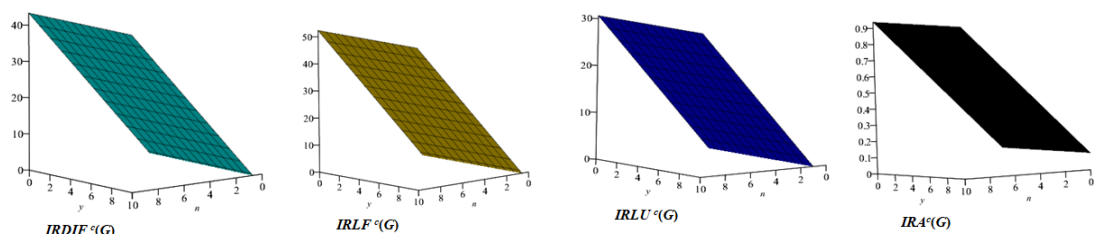
$$\begin{aligned}
&= 2 \left[\sum_{n_1 n_2 \in E_{2,2}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} + \sum_{n_1 n_2 \in E_{2,4}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} + \sum_{n_1 n_2 \in E_{4,4}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} \right. \\
&+ \sum_{n_1 n_2 \in E_{4,6}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} + \sum_{n_1 n_2 \in E_{6,6}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} + \sum_{n_1 n_2 \in E_{6,8}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} \\
&+ \left. \sum_{n_1 n_2 \in E_{8,8}^c} \frac{|\tau_G(n_1) - \tau_G(n_2)|}{(\tau(n_1) + \tau(n_2))} \right] \\
&= 2 \left[(2) \frac{|2-2|}{(2)+(2)} + (4) \frac{|2-4|}{(2)+(4)} + (4) \frac{|4-4|}{(4)+(4)} + (8n-4) \frac{|4-6|}{(4)+(6)} + (8n-14) \frac{|6-6|}{(6)+(6)} \right. \\
&+ (8n-16) \frac{|6-8|}{(6)+(8)} + (6n^2 - 24(n-1)) \frac{|8-8|}{(8)+(8)} \left. \right] \\
&= 2 \left[(4) \frac{|-2|}{6} + (8n-4) \frac{|-2|}{10} + (8n-16) \frac{|-2|}{14} \right] \\
&= (2) \left(\frac{288n-264}{105} \right)
\end{aligned}$$

$$(vii) \quad IRGA(^cG) = \sum_{n_1 n_2 \in E(G)} \ln \frac{\tau_{n_1} + \tau_{n_2}}{2(\sqrt{\tau_{n_1} \tau_{n_2}})}$$

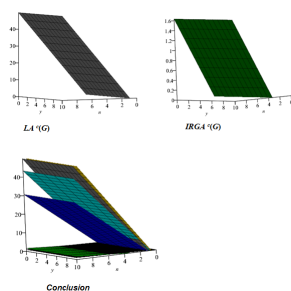
$$\begin{aligned}
&= \sum_{n_1 n_2 \in E_{2,2}^c} \ln \frac{\tau_{n_1} + \tau_{n_2}}{2(\sqrt{\tau_{n_1} \tau_{n_2}})} + \sum_{n_1 n_2 \in E_{2,4}^c} \ln \frac{\tau_{n_1} + \tau_{n_2}}{2(\sqrt{\tau_{n_1} \tau_{n_2}})} + \sum_{n_1 n_2 \in E_{4,4}^c} \ln \frac{\tau_{n_1} + \tau_{n_2}}{2(\sqrt{\tau_{n_1} \tau_{n_2}})} \\
&+ \sum_{n_1 n_2 \in E_{4,6}^c} \ln \frac{\tau_{n_1} + \tau_{n_2}}{2(\sqrt{\tau_{n_1} \tau_{n_2}})} + \sum_{n_1 n_2 \in E_{6,6}^c} \ln \frac{\tau_{n_1} + \tau_{n_2}}{2(\sqrt{\tau_{n_1} \tau_{n_2}})} + \sum_{n_1 n_2 \in E_{6,8}^c} \ln \frac{\tau_{n_1} + \tau_{n_2}}{2(\sqrt{\tau_{n_1} \tau_{n_2}})} \\
&+ \sum_{n_1 n_2 \in E_{8,8}^c} \ln \frac{\tau_{n_1} + \tau_{n_2}}{2(\sqrt{\tau_{n_1} \tau_{n_2}})} \\
&= 2 \ln \frac{2+2}{2\sqrt{(2)(2)}} + 4 \ln \frac{2+4}{2\sqrt{(2)(4)}} + 4 \ln \frac{4+4}{2\sqrt{(4)(4)}} + (8n-4) \ln \frac{4+6}{2\sqrt{(4)(6)}} \\
&+ (8n-14) \ln \frac{6+6}{2\sqrt{(6)(6)}} + (8n-16) \ln \frac{6+8}{2\sqrt{(6)(8)}} + (6n^2 - 24(n-1)) \ln \frac{8+8}{2\sqrt{(8)(8)}} \\
&= 4 \ln \frac{2+4}{2\sqrt{(2)(4)}} + 8n-4 \ln \frac{4+6}{2\sqrt{(4)(6)}} \\
&+ (21q-14) \left(\ln \frac{r+2+r+2}{2(\sqrt{(r+2)^2})} \right) + (8n-16) \ln \frac{6+8}{2\sqrt{(6)(8)}} \\
&= 4 \left(\ln \frac{3}{2\sqrt{3}} \right) + (8n-4) \left(\ln \frac{5}{\sqrt{24}} \right) + (8n-16) \left(\ln \frac{7}{\sqrt{48}} \right)
\end{aligned}$$

n	$IRLU^c(G)$	$IRA^c(G)$	$IRDIF^c(G)$	$IRLF^c(G)$	$LA^c(G)$	$IRGA^c(G)$
(3)	6.7585	0.3552	10.6666	13.3029	11.4285	23.4254
(4)	10.1643	0.4384	15.333	18.8781	16.9142	31.1507
(5)	13.5701	0.5216	20	24.4535	22.4	39.5904
(6)	16.979	0.6048	24.6666	30.0829	27.8857	47.6729
(7)	20.3817	0.688	29.9999	35.6043	33.3714	55.7553
(8)	23.7875	0.7714	34	41.1796	38.8517	63.8378
(9)	27.1933	0.8544	38.6666	46.7551	44.3428	71.9203
(10)	30.9591	0.9376	43.3333	52.3305	49.8285	80.0028
(11)	34.0049	1.028	48	57.9058	55.3142	88.0852
(12)	37.4107	1.104	52.66	63.4812	60.8	96.1677

Table 7: Table for connection based irregularity indices of *RHOX*



Comparison The connection based irregularity indices $IRLU^c(G)$, $IRA^c(G)$, $IRDIF^c(G)$, $IRLF^c(G)$, $LA^c(G)$, $IRGA^c(G)$ presented by blue, black, cyan, yellow, gray and green colours. Moreover, $IRGA^c(G)$ is dominant connection based irregularity index for rhombus silicate.



4. Conclusion

In the current article, we found connection based irregularity indices for *RHSL* and *RHOX*. The obtained results are also illustrated with help of tables consisting of numerical value and figures consisting of graphical presentation.

- From the computed connection based irregularity indices rhombus silicate *RHSL* and dominant values than *RHOX* rhombus oxide.
- It is found that among all the computed connection based irregularity indices $IRGA^c(G)$ is most dominant and steady for rhombus silicate.

Acknowledgement

The authors are fully agreed to the valuable suggestions and remarks of the referees that supported this manuscript in its nurture.

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