

MHD effect on fractional Casson nanofluid with constant boundary conditions and generalized Mittag-Leffler Kernel

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Abstract

Transport thermal fact of Casson flow explored by using a new analytical solution which is established on Prabhakar's fractional approach modified by Fourier and Fick's law. The constitutive relation used in the governing equations, and the Laplace transform technique used to obtain analytical solutions. The solution for temperature and velocity field analyzed by using Mathcad software's graphical representation. Fluid characteristics revealed a variety of aspects for various flow parameters and fractional parameter values to produce significant results. As a result, rise the values of fractional parameters can improve fluid properties, which is useful in some experimental data fitting in the heating and cooling phenomena, especially in electronic devices.

Keywords: Magnetohydrodynamics; Nano fluids, Casson fluid, Prabhakar fractional derivative, MHD.

Mathematics Subject Classification: 2010 : 62C86; 90B50; 91B06; 03B52; 62A86.

1. Introduction

Nanotechnology has recently sparked the interest of many researchers through its various applications in industry. Choi [1] was the first researcher to introduce nanofluids in 1995, and he also described micron-scale particles smaller than 100 nm in diameter. He investigated stretched sheets with heat transport properties and various uses of nano-fluids in scientific field like cancer therapy. Ali et al. [2] A Maxwell nanofluidic flow is introduced through a stretched sheet due to chemical reactions with multi-slip properties. Air purification and heat recovery are discussed in a recent portfolio by Liu Yang et al. [3]. In applied mathematics, the fractional derivative is the derivative of real or complex number. Firstly, it appeared in 1965 by Gottfried Wilhelm Leibniz to Guillaume de Hôpital Ortigueira et al. [4]. At the same time, Leibniz describe the similarity between the binomial theorem and the Leibniz rule for the fractional derivative of the product of two functions. The paper can find all the elements the idea of fractional integral and differential, the reciprocal relationship between them, the understanding of fractional differential and integral can be regarded as the same generalized operation, even the unification of any real-order differential and integral symbol. Evans et al. [5]. Independently, Liouville laid the foundation of the discipline in a paper in 1832. Abel et al. [6]. The self-study Oliver Heaviside introduced the practical application of fractional differential operators in the analysis of transmission lines in about 1890. Niels et al. [7]. The theory and application of fractional calculus have been greatly expanded in the nineteenth

and twenty centuries. Many contributors have given the definition of fractional derivatives and integrals. Liouville et al. [8].

Zheng et al. [9] discussed a generalized Oldroyd-B fluids having fractional derivative has slip impacts on MHD flow. Using the generalized Fourier's law, a fractional model for MHD flow of Casson fluid with nanoparticles was developed by Sheikh et al. [10]. Saqib et al. [11] introduced a ramping heating, the shape effect on MHD fluid of a time fractional FerroBrinkman type nano-fluid. Ezzat [12] deliberated non-Newtonian thermoelectric MHD fluid having fractional derivative heat exchange. Akgul and Siddique [13] observed fractal-fractional differential derivative are used to analyze MHD Couette flow. For the generalized MHD equations, commutator estimations involving fractional derivatives and local presence by Jiang et al. [14]. Haq et al. [15] debated MHD flow of a generalized second-grade fluid employing fractional Caputo-Fabrizio derivatives and modified Darcy's equation for exponential heating.

Casson fluid, commonly well known as a shear thinning fluid, which is one of several non-Newtonian fluids. It acts, in two different states matter. When the applied yield strength is larger than the shear strength, the response takes on the qualities of a solid, and when the applied yield strength is less, the response takes on the properties of a liquid. Examples include blood human, honey, jelly, tomato sauce and jelly, among others. Casson fluid are investigated by mathematicians, biomedical field, scientists, and engineers due to their vast range of uses [16, 17]. Sheikh et al. [18] A novel fractional Casson fluid model based on extended Fick and Fourier's laws and heat and mass transfer is discussed. Khan et al. . Khan et al. [19] conducted an infinite plate causes an un-steady fluid of a generalized Casson flow with fractional derivative. Arif et al. [20] analyzed how non-Newtonian fluids can be used to solve engineering difficulties with increased temperature, Casson fluid has taken. Ali et al. [21] deliberated Wright solution fractional time function Casson flow in free convection Using a fractional derivative technique, thermal activity of typical Casson nano-particles with ramping temperature because of infinite vertical plate was studied by Raza et al. [22]. Saqib et al. [23] Observed BTF nonlocal fractional model containing mixed nano-particles with symmetric MHD channel flow. Based on the simplified Fourier Law, a Prabhakar fragmentary technique for the convection fluid of Casson liquid along an oscillating plate discussed by Sarwar et al. [24]. Wang et al. [25] observed unsteady heat transport flow of Casson nanofluids having Prabhakar's type generalised Mittag-Leffler kernel.

The Prabhakar integral was obtained by including the three-parameter Mittag-Leffler function in the kernel of the Riemann-Liouville integral derivative. Prabhakar derivative was obtained by replacing the Riemann Liouville integral. The Prabhakar fractional function is used to describe non-linear and non-local substances that have strange tendencies in disordered persons [26, 27]. Elnaqeeb et al. [28] investigated carbon nano-tube nano-fluids flow naturally with Prabhakar-like thermal transportation. Shah et al. [29] debated Natural convection flow with simplified thermal transfer in Prabhakar-like fractional Maxwell fluid. The Prabhakar function, also known as the three-parameter Mittag-Leffler function, is a theory and application of the Mittag-Leffler function expressed by Garra and Garrappa [30]. Asjad et al. [31] discussed advancement of a nonNewtonian fluid in a convection channel with hybrid nanoparticles and Prabhakar fractional derivative - analytical method. Basit et al. [32] analyzed convective flow of a fractional second-grade fluid contain various nanoparticles with Prabhakar fractional derivative and non-uniform velocity at the edge. In the sense of Hilfer-Prabhakar derivatives, a new analytical method to solve variable order fractional integro differential equation deliberated by Sene and Srivastava [33]. Fractional differential equation with generalised Mittag-Leffler input consistency expressed by Derakhshan [34].

2. Solution of Temperature

Laplace transform of equation (10),

$$a_4 \text{Prs} \bar{T}(y, s) = -\bar{q}_y(y, s). \quad (2.1)$$

taking Laplace transform of equation (16),

$$\bar{q}_y(y, s) = -a_5 s^\beta (1 - as^{-\alpha})^\gamma \bar{T}_y(y, s). \quad (2.2)$$

Apply Laplace transform of boundary conditions,

$$\bar{T}(y, 0) = 0, \bar{T}(0, s) = \frac{1}{s}, y \rightarrow \infty \rightarrow \bar{T}(y, s). \quad (2.3)$$

Taking derivative of equation (19),

$$\bar{q}_y(y, s) = -a_5 s^\beta (1 - as^{-\alpha})^\gamma \bar{T}_{yy}(y, s). \quad (2.4)$$

Combine the equations then get the final result,

$$\bar{T}_{yy}(y, s) = \frac{a_4}{a_5} \frac{\text{Prs}}{s^\beta (1 - as^{-\alpha})^\gamma} \bar{T}(y, s). \quad (2.5)$$

$$\bar{T}_{yy}(y, s) = a_7 \frac{\text{Prs}}{s^\beta (1 - as^{-\alpha})^\gamma} \bar{T}(y, s). \quad (2.6)$$

Apply boundary conditions on equation (23) the solution form of the equation,

$$\bar{T}(y, s) = \frac{1}{s} e^{-y \sqrt{\frac{a_7 \text{Prs}^{1-\beta}}{1 - as^{-\alpha}}}}. \quad (2.7)$$

Now, we simplify the solution

$$\bar{T}(y, s) = 1 + \sum_{p=1}^{\infty} \sum_{q=0}^{\infty} \frac{(-y \sqrt{\text{Pra}_7})^p}{p!} s^{\frac{p(1-\beta)-1-\alpha q}{2}} \alpha^q \frac{\Gamma(\frac{\gamma p}{2} + 1)}{\Gamma(\frac{\gamma p}{2} + 1 - q)}. \quad (2.8)$$

Applying the Laplace inverse transform method equation(25),

$$T(y, t) = 1 + \sum_{p=1}^{\infty} \sum_{q=0}^{\infty} \frac{(-y \sqrt{\text{Pra}_7})^p}{p!} t^{\frac{p(1-\beta)-2-\alpha q}{2}} \alpha^q \frac{\Gamma(\frac{\gamma p}{2} + 1)}{\Gamma(\frac{\gamma p}{2} + 1 - q) \Gamma(\frac{p(1-\beta)}{2} - 1 - \alpha q)}. \quad (2.9)$$

Solution of Diffusion Equation Taking Laplace transform of equation (12),

$$\text{Scs} \bar{C}(y, s) = -\bar{J}_y(y, s). \quad (2.10)$$

Applying Laplace transform of equation (17),

$$\bar{J}(y, s) = -a_6 s^\beta (1 - as^{-\alpha})^\gamma \bar{C}_y(y, s). \quad (2.11)$$

Boundary conditions

$$y \rightarrow \infty \rightarrow \bar{C}(y, s) = 0, \bar{C}(0, s) = \frac{1}{s} \quad (2.12)$$

Combine the both equations

$$\bar{C}_{yy}(y, s) = \frac{a_6^{-1} \text{Scs}}{s^\beta (1 - as^{-\alpha})^\gamma} \bar{C}(y, s). \quad (2.13)$$

appropriate form of equation,

$$\bar{C}(y, s) = s^{-2} e^{-y \sqrt{\frac{a_6^{-1} Scs}{s^{\beta}(1-as^{-\alpha})^{\gamma}}}} \quad (2.14)$$

Now, Applying the Laplace inverse transform.

First we apply the series method

$$\bar{C}(y, s) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} s^{-2+\frac{n}{2}(1-\beta)-\alpha m} \alpha^m \frac{\Gamma \gamma_{\frac{n}{2}}^n + 1}{\Gamma \gamma_{\frac{n}{2}}^n + 1 - m} \frac{(-y \sqrt{a_6^{-1} Sc})^n}{n!}, \quad (2.15)$$

$$C(y, t) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \alpha^m \frac{s^{-3+\frac{n}{2}(1-\beta)-\alpha m}}{\sqrt{-2+\frac{n}{2}(1-\beta)-\alpha m}} \frac{(\Gamma_{\frac{n}{2}}^n + 1)}{\Gamma(\gamma_{\frac{n}{2}}^n + 1 - m)} \frac{(-y \sqrt{a_6^{-1} Sc})^n}{n!}. \quad (2.16)$$

3. Solution of Fluid Velocity

Taking Laplace Transform of velocity equation

$$a_8 s \bar{u}(y, s) - \bar{u}_{yy}(y, s) - a_9 [\text{Gr} \bar{T}(y, s) + \text{Gm} \bar{C}(y, s)] + \text{AM} u(y, s) = 0 \quad (3.1)$$

Boundary conditions

$$\bar{u}(0, s) = \frac{1}{s}, \quad \bar{u}(y, s) = 0, \text{ as } y \rightarrow \infty \quad (3.2)$$

With the help of equation (24) and (31),

$$\bar{u}_{yy}(y, s) - a_8 s \bar{u}(y, s) - \text{AM} \bar{u}(y, s) = a_9 \left[\text{Gr} \frac{1}{s} e^{-y \sqrt{\frac{a_7 \text{Pr} s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}}} + \text{Gm} \frac{1}{s} e^{-y \sqrt{\frac{a_6^{-1} Sc s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}}} \right] \quad (3.3)$$

Applying boundary conditions on (36),

$$\begin{aligned} \bar{u}(y, s) = \frac{1}{s} e^{-y \sqrt{a_8 s + \text{AM}}} + \frac{a_9 \text{Gr} (1 - as^{-\alpha})^{\gamma}}{sa_7 \text{Pr} s^{1-\beta} - (a_8 s + \text{AM})(1 - as^{-\alpha})^{\gamma}} \left[e^{-y \sqrt{(a_8 s + \text{AM})}} - e^{-y \sqrt{\frac{a_7 \text{Pr} s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}}} \right] \\ - \frac{a_9 \text{Gm} (1 - as^{-\alpha})^{\gamma}}{sa_6^{-1} Sc s^{1-\beta} - (a_8 s + \text{AM})(1 - as^{-\alpha})^{\gamma}} \left[e^{-y \sqrt{(a_8 s + \text{AM})}} - e^{-y \sqrt{\frac{a_6^{-1} Sc s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}}} \right] \end{aligned} \quad (3.4)$$

The Laplace inverse of this velocity equation is so difficult so it solved by algorithm method.

4. Analysis of results and discussion

This segment presents the results as enumerated from the above mentioned coding for the varying behaviors of fluid velocity, temperature, concentration function, which are observed and tabulated for apposite ranges of the sundry parameters. The estimates of the present finding are computed by putting the values of involved parameter to be fixed as: $\text{Pr}=15$, $\text{M}=0.2$, $\text{Sc}=0.2$, $\text{Gm}=0.1$, $a=1$, $\text{Gr}=9$, $t=1$. Figure 2 is comparison graph of base paper.[30] When we take magneto hydrodynamics and concentration zero then the velocity layers overlap each other. Figures 3, 4 and 5 shows the fractional parameter of α, β, γ Which control the flow behavior of fluid. For greater value of β, γ, α the fluid properties increase so boundary layer thickness increase. Now for Figure 9 and 14 this graph based on Schmidt number Sc respectively.

If we increase the value of Schmidt number then the boundary layer thickness decrease and behavior of graph decreased. Figures 6 and 7 ϕ is a fractional parameter. Heat conduction increases with rise value of ϕ , the heat transfer increases, which causes rise temperature profile. Figures 8 and 10 based on for different values of Pr. If we increase the value of Pr then the thickness of thermal boundary layer will be decreased and velocity layer will be decreased. Figure 11 is based on for different values of M. Velocity decreased with higher value of M. Greater value of M relate to greater resistive type force so velocity reduce. It is found that boundary layer thickness decreases. Figure 12 is based on for different values of Gr. Shows the effect of Gr on the velocity distribution. The ascent value of Gr results in an increase in buoyancy and thus an increase in fluid velocity. Velocity of boundary layer thickness also increases. F13 based on different values of Gm. Gm represent same as mass groshaf number. It has same physical significant as Gr. The impact of Gm on the velocity division is displayed. Increase in Gm causes an increase buoyancy forces due which the fluid velocity increases. Velocity of boundary layer thickness also increases.

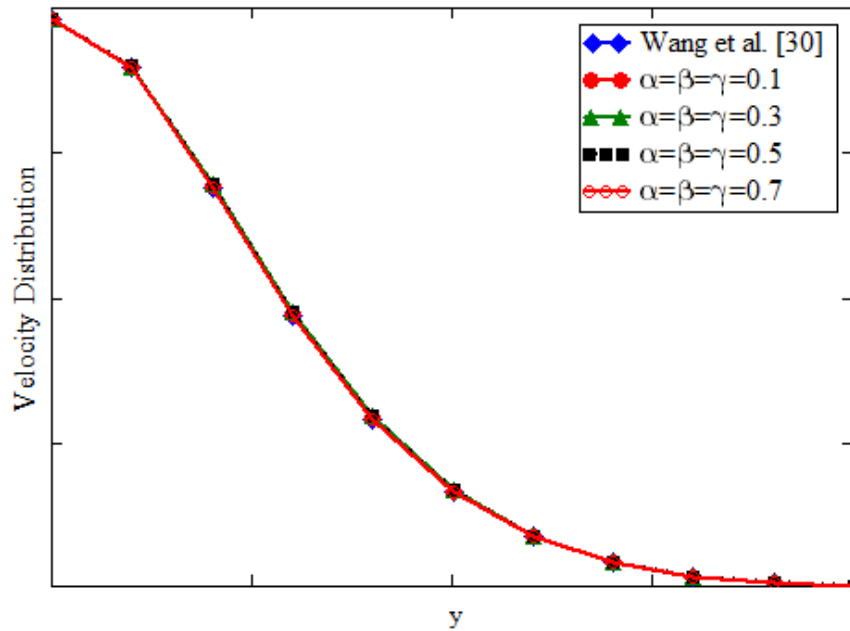


Figure 4.1: Sketch of comparison graph of Wang et al.[30] We take $M=0$ and Concentration=0. When $t=1$, $Gr=3$, $Pr = 6.2$.

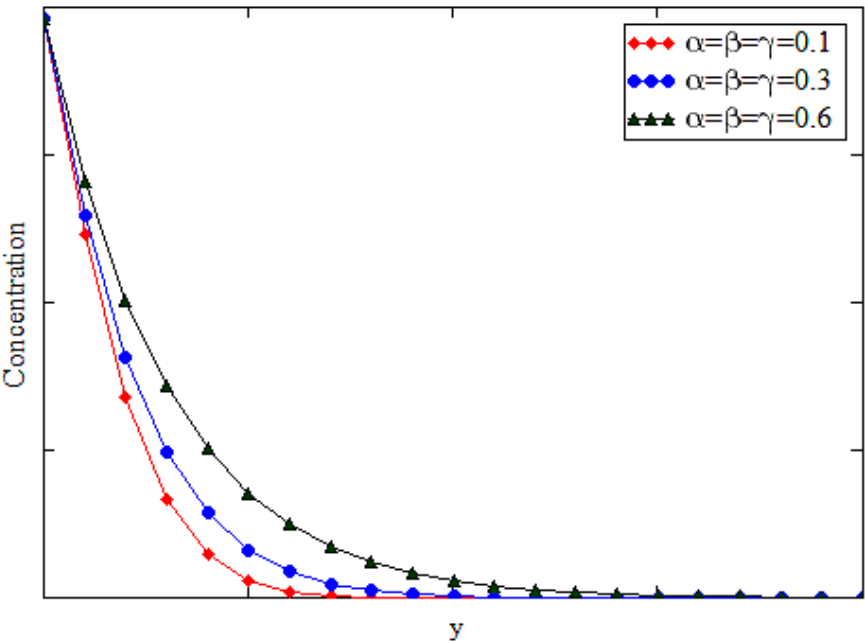


Figure 4.2: Sketch for different inputs of β, γ, α . When $Pr=6.2, t=1, Sc=1$.

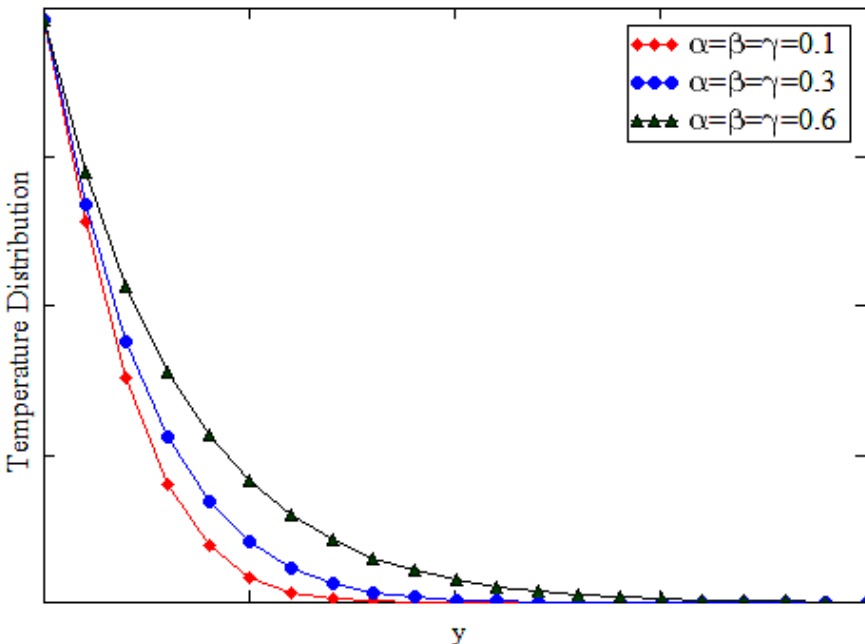


Figure 4.3: sketch for different inputs of α, β, γ . When $Pr=6.2, t=1, Sc=1$.

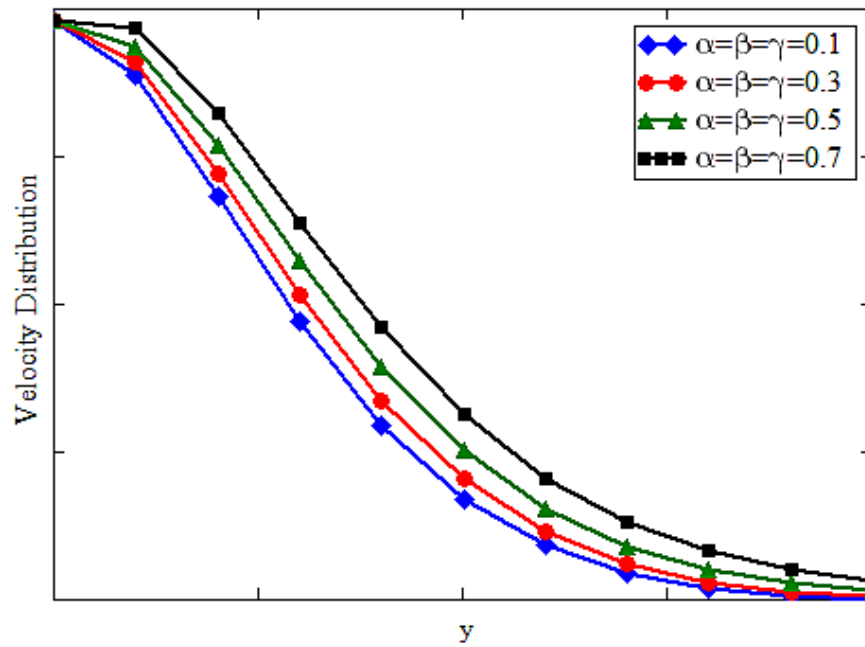


Figure 4.4: Sketch for different inputs of α, γ, β . When $Pr=6.2$, $t=1$, $Sc=1$.

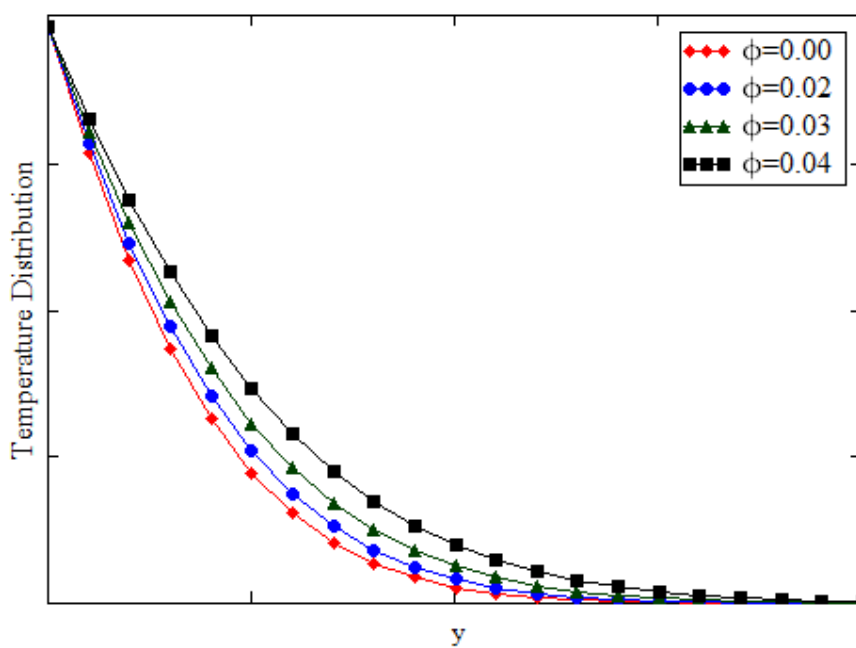


Figure 4.5: Sketch for different inputs of ϕ . When $t=0.2$, $Sc=1$, $Pr=6.2$.

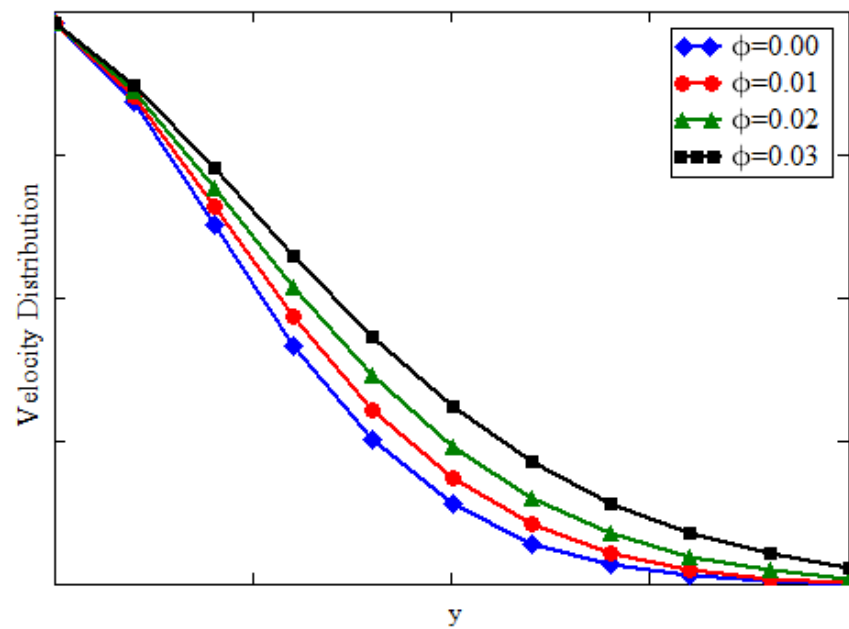


Figure 4.6: Sketch for different inputs of ϕ . When $Sc=1$, $t=0.2$.

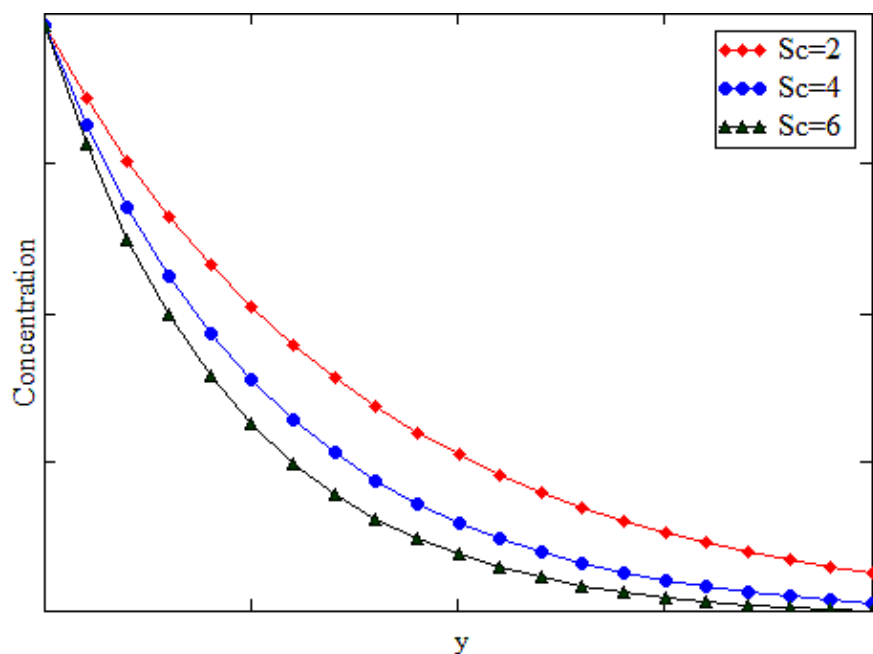


Figure 4.7: Sketch for different inputs of sc . When $Pr=6.2$, $t=0.2$.

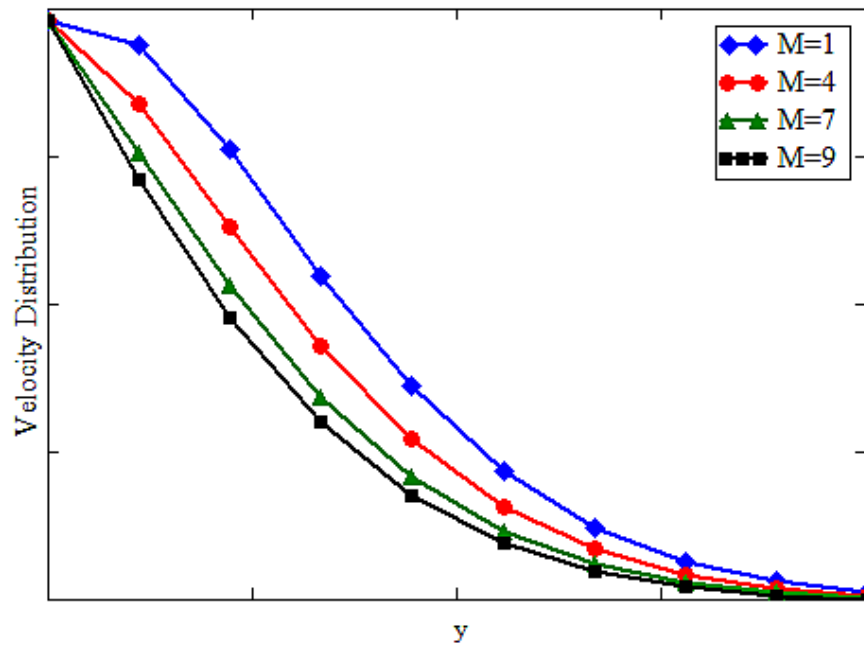


Figure 4.8: Sketch for different inputs of M . When $Pr=6.2$, $Gr=9$, $t=0.2$.

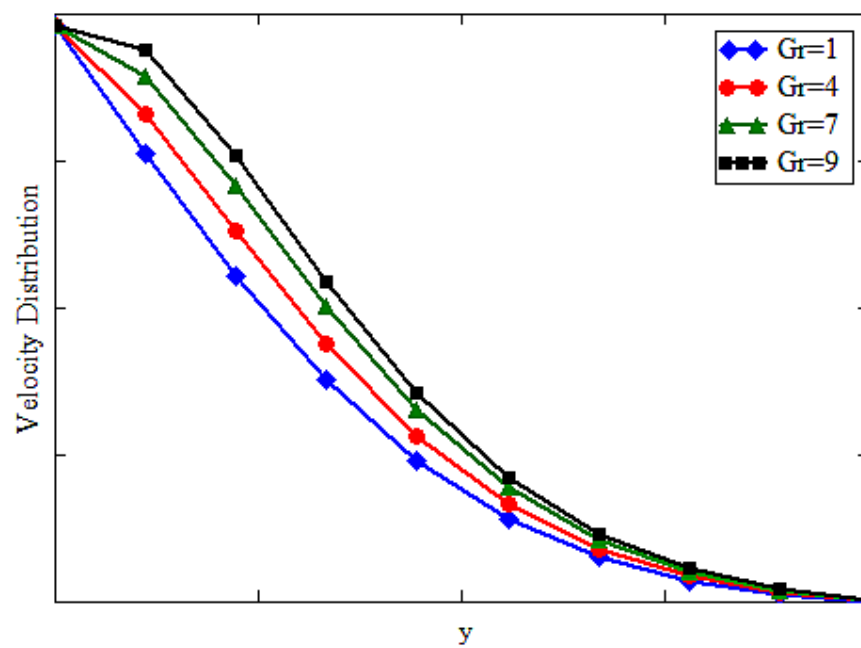


Figure 4.9: Sketch for different inputs of Gr . When $Pr=6.2$, $t=0.2$, $Sc=2$.

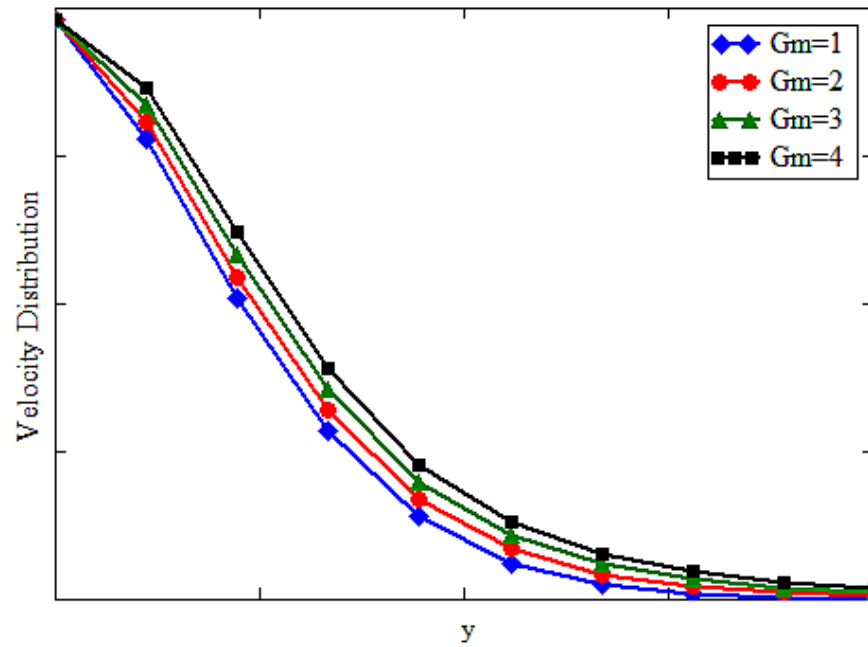


Figure 4.10: Sketch for different inputs of Gm . When $Pr=6.2$, $t=0.2$, $Sc=1$.

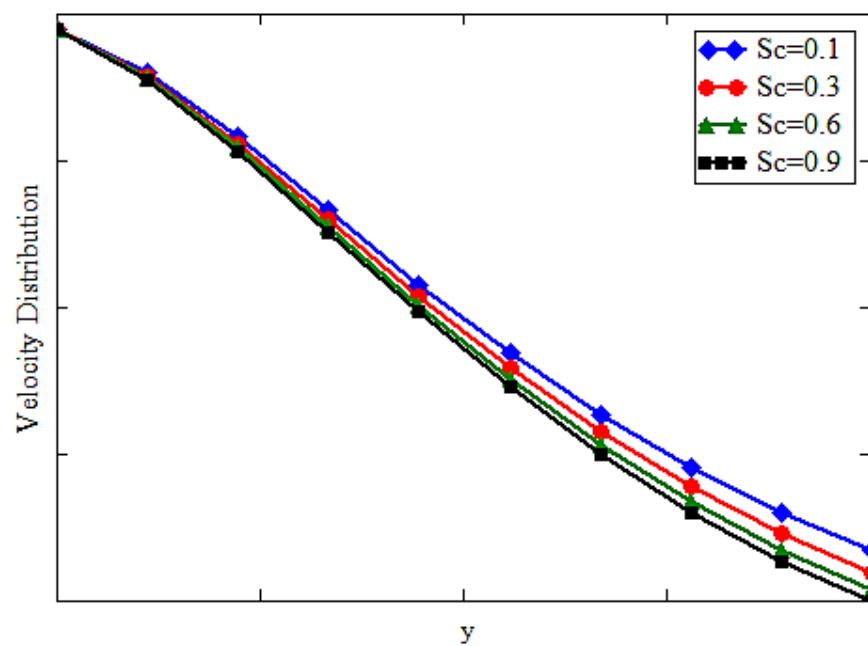


Figure 4.11: Sketch for different inputs of Sc . When $t=1$, $Pr=6.2$, $Gr=3$.

5. Conclusions

This article's main objective is to propose the un-steady heat conduction fluid of Casson nano-fluid with modified Mittag-Leffler formulas. This paper provides new research. We use the Prabhakar fractional derivative along the modified Fick law to introduce the fractional diffusion equation in this model. The flow characteristics of nano-fluid were explained analytically using the Laplace transform. The following are some of the results we found through graphs:

- The velocity reduces against parameters of Pr , and M .
- Velocity increases against parameter Gr and ϕ .
- Flow properties of concentration, temperature and velocity have greater value by rise the values of fractional parameter γ, α, β .

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